DYNAMIC ANALYSIS
OF
SHEARWALL STRUCTURES

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

BY

DARSHAN SINGH SAHOTA

to the

DEPARTMENT OF CIVIL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY 1972

CERTIFICATE

Certified that the thesis entitled "Dynamic Analysis of Shearwall Structures" is the bonafide work done by Mr. D.S. Sahota under my guidance and has not been submitted elsewhere for the award of a Degree.

(Y.C. DAS)
PROFESSOR AND HEAD
DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY
KANPUR - 16

ACKNOWLEDGEMENT

The author wishes to express the deep sense of gratitude to Dr. Y.C. Das for proposing the problem and continued expert guidance. Sincere thanks are due to Dr. D.V. Mallick for whole-hearted cooperation and supervision in conducting the experimental work at I.I.T., New Delhi. The author is highly grateful to Dr. M.P. Kapoor for discussion and valuable suggestions.

Thanks are due to Mr. M. Hariharan, Research Scholar, Mr. K. Natarajan of Computer Centre IIT/Kanpur and other colleagues for their kind help in carrying out this work.

The author thanks M/s. V.K. Saxena and S.N. Pandey for efficient typing of the manuscript.

ABSTRACT

Dynamic analysis of two types of shearwall structures has been carried out by two methods using stiffness approach. The shearwall structure Type I comprises of a shearwall with a one bay rigid frame on either side while Type II consists of two shearwalls connected at floor levels. In method I, axial deformations in the columns or shearwalls have been neglected while in method II these have also been considered along with bending effects. The response of the structures subjected to wind and earthquake loading has been determined by modal analysis which necessitated the determination of the natural frequencies and mode shapes of the structures. The results obtained by both the methods were compared and conclusions have been drawn. The models of the above structures were excited harmonically and the experimental values of natural frequencies have been compared with the theoretical values.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

INTRODUCTION

In tall buildings, it becomes very important to ensure adequate lateral stiffness to resist the lateral loads which may arise due to mind, seismic or for that matter even blast effects. The general term 'blast' refers to both vibrations induced in the soil and to fluctuations of air pressure due to man-made explosions. Blast effects due to soil vibrations may be considered as seismic excitations. The provision of shearwalls to achieve such rigidity has been found to be very effective and economical. Furthermore in tall structures, it is necessary to know their dynamic response subject to these time-dependent loads besides their response to static loads. As a result of the effect of these time dependent loads, forces of inertia are developed in the structural elements of the buildings. If the individual members are not sufficiently strong, they suffer substantial displacements and undergo cracks making the structure as a whole unfit for further use. Under the action of wind, a tall building

will be continually buffeted by gusts and other aerodynamic forces Although the structure will tend toward a mean position, it will oscillate continuously. It has been observed that this oscillating motion will occur primarily at the fundamental period of vibration of the building (1,2). One of the methods of dynamic analysis requires that the infinite degree - of - freedom continuous structure be idealised as a finite degree-of-freedom discrete structure, assembly of stiffness, mass and damping coefficient matrices of the structure and the use of digital computers to seek the solution.

A lot of literature (3,4,5) is available on the static analysis of tall buildings but the available literature on the dynamic analysis is relatively limited. In 1964, Clough, Wilson and King [6] presented an efficient method of static analysis for coupled shearwalls, frames and combinations of frames and shearwalls. Symmetric arrangement of shearwalls in the floor plan and rigid floor translations without rotations were assumed. Shearwall arrangements in the various parallel frames might be specified arbitrarily but the assumption that the building deflected without twisting was consistent only with a

reasonably symmetric distribution of stiffnesses. Shearwalls were considered as columns with finite width and their effects on girder end rotations were included. The lateral stiffness matrix with one degree of freedom per storey corresponding to the lateral displacement at each storey was established for each plane frame parallel to each other by tri-diagonalisation procedure. The stiffness matrices were superposed to obtain the total building stiffness matrix which provided the lateral displacements as the solution of equilibrium equations. In 1966, Webster[7] presented a stiffness method of analysis of shearwalls with frames, considering the shearwalls as deep columns. The method was much similar to that proposed by Clough, Wilson and King[6], however, Webster had suggested the extension of this method to carry out the dynamic analysis of frames. In this method, the floor was treated as fully rigid in-plane (i.e. all in-plane floor movements were defined by rigid body deformations). This device significantly reduces the order of formulation and enhances the numerical accuracy. Webster developed a lateral stiffness matrix which relates the lateral loads applied at each storey level with the storey level displacements.

Jenkins and Harrison[8] developed methods of analysis of tall buildings with shearwalls under bending and torsion. They suggested a stiffness method for the bending analysis and an energy method for the torsion analysis. The stiffnesses have been tabulated for two types of shearwall structures, which can be arranged to give a stiffness matrix and finally by partitioning technique, the stiffness matrix can be reduced to a lateral stiffness matrix. Here more degrees of freedom per storey have been taken as compared to the method presented by Webster[7].

In the present work, two types of 10-storey shearwall structures (Fig. 1.1) have been analysed by using stiffness approach based on references (7) and (8). In chapters II and III, the development of stiffness matrices has been illustrated. The general stiffness matrix is reduced to a lateral stiffness matrix as given in Chapter IV. Chapter V illustrates the formulation of dynamic problem and the method to find frequencies, time periods and modes of vibration of a multidegree freedom system. In Chapter VI, modal analysis is used to analyse the structure against wind and earthquake loading. In Chapter VII, two numerical examples have been solved to illustrate the application of analytical

method, in determining the dynamic behaviour of shearwall structures. The results by both the methods have been compared.

Chapter VIII deals with the experimental set-up and testing of models of shearwall structures. The models were excited harmonically to determine the natural frequencies and mode shapes.

In Chapter IX conclusions have been drawn and suggestions have been made for future work in this field.

FIG. 14

DYNAMIC ANALYSIS OF SHEARWALL STRUC

(a). SHEARWALL STRUCTURE TYPE I

(b). " " " II

(c). MODEL TO DETERMINE NATURAL FREQUENCIES & MODE SHA

CHAPTER II

DEVELOPMENT OF THE STIFFNESS MATRIX FOR SHEARWALL STRUCTURES - METHOD I

The method I is based on the construction of a general stiffness matrix for a shearwall structure, and its subsequent reduction, by matrix partitioning methods, to form a lateral stiffness matrix (Chapter IV) which directly relates the lateral displacements to the applied loads at each storey level. It is assumed that the floor slabs translate and rotate as rigid bodies.

2.1 BASIC ASSUMPTIONS :

a) The structure is assumed to be perfectly elastic and subject only to small deformations.

b) Flexural deformations only are considered.

c) All joints are rigidly connected.

d) The floor slabs at each storey level are infinitely rigid in their own plane, but have

no stiffness normal to that plane. They may thus be assumed to translate and rotate as rigid bodies.

e) The proportions of the shearwalls are such that plane sections remain plane.

f) Shearwalls and columns extend continuously from base to top and beams from side to side.

g) The proportions of the beams are such that their action may be taken as that of line elements. The finite widths of shearwalls and columns are however taken into account.

2.2 BASIC SLOPE DEFLECTION EQUATIONS FOR A UNIFORM MEMBER :

If, for a shearwall, plane sections may be assumed to remain plane, it follows that the action of the member will be equivalent to that of a uniform line element having the same stiffness, placed along its centroidal axis. The fact that such a shearwall has finite width must, however, affect the structural action of the beams framing into the wall. This effect will be taken to be analogous to that produced by rigid end gussets on the beams, the lengths of

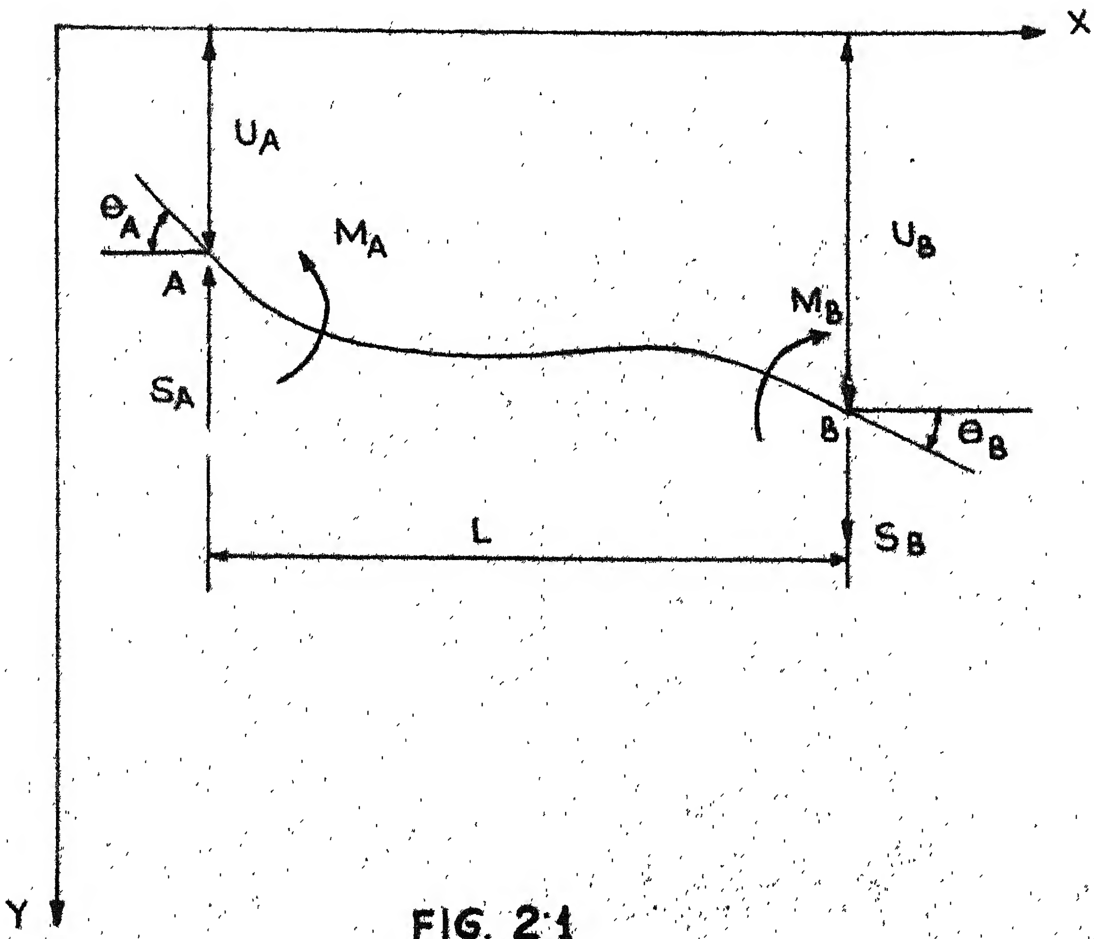


FIG. 2·1

SIGN CONVENTION FOR END LOADS & DISPLACEMENTS

which are equal to the distances from the ends of the beam to the centroidal axis of the walls to which they are attached.

Consider first a uniform member as shown in Fig. 2.1. The member is subjected to end moments and shears which cause the end rotations and displacements shown. From normal slope deflection theory, it is apparent that

$$M_A = -\frac{4EI}{L}\theta_A - \frac{2EI}{L}\theta_B - \frac{6EI}{L^2}(U_A - U_B) \tag{2.1}$$

$$M_B = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B + \frac{6EI}{L^2}(U_A - U_B) \tag{2.2}$$

$$S_A = S_B = -\frac{6EI}{L^2}\theta_A - \frac{6EI}{L^2}\theta_B - \frac{12\ EI}{L^3}(U_A - U_B) \tag{2.3}$$

2.3 UNIFORM MEMBER WITH RIGID END GUSSETS :

Consider now the member shown in Fig. 2.2 which represents a typical beam with rigid end gussets. A rotation θ_B is given to the end B, as shown, by application of end moments and shears. The stiffness coefficients $\bar{\bar{K}}_B$ and $\bar{\bar{K}}_c$ are defined to be such that

$$M_A = \bar{\bar{K}}_c\ \frac{EI}{L}\theta_B \text{ and } M_B = \bar{\bar{K}}_B\frac{EI}{L}\theta_B$$

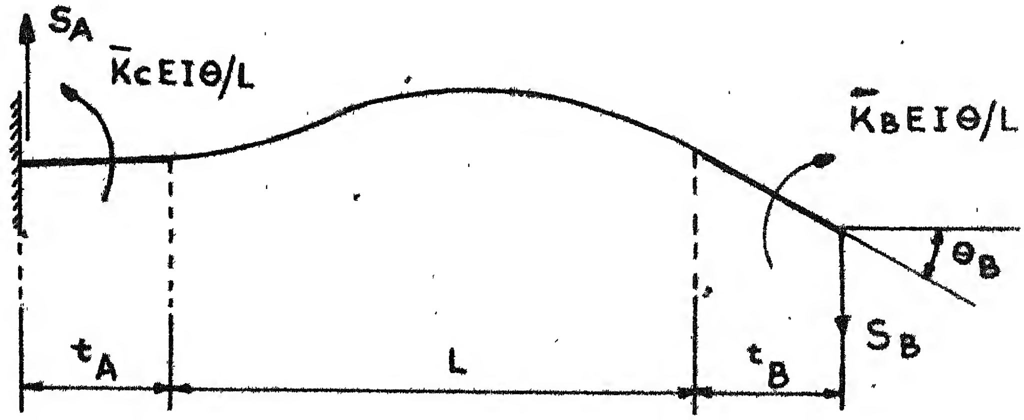


FIG. 2:2

UNIFORM MEMBER WITH RIGID END GUSSETS.

Application of a rotation θ_A at end A would similarly define the coefficient $\bar{K}_A$. From Fig. 2.2 it is apparent that, by taking moments about the inner ends of the gussets, the following relationships may be obtained:

$$\bar{K}_B \frac{EI}{L} \theta_B + S_B . t_B = \frac{4EI}{L} \theta_B + \frac{6EI}{L^2} t_B \theta_B$$

$$\bar{K}_c \frac{EI}{L} \theta_B - S_A . t_A = - \frac{2 EI}{L} \theta_B - \frac{6EI}{L^2} t_B \theta_B .$$

And, taking moments about end A, it follows that

$$\bar{K}_c \frac{EI}{L} \theta_B - \bar{K}_B \frac{EI}{L} \theta_B = S_B (L + t_A + t_B) .$$

In the above expressions, there is no reference to relative lateral displacement between ends A and B, which is precluded by the assumption that the axial shortening of the columns may be neglected. Now the overall lateral equilibrium of the member demands that $S_A = S_B$ and hence these terms may readily be eliminated from the above equations to give the following results :

$$\bar{K}_B = 4 + 12 \frac{t_B}{L} (1 + \frac{t_B}{L}), \qquad (2.4)$$

$$\bar{K}_c = 2 + \frac{6}{L}\left(t_A + t_B + \frac{2\, t_A\, t_B}{L}\right) \tag{2.5}$$

The coefficient $\bar{K}_A$ may be obtained in a very similar manner, the final result being

$$\bar{K}_A = 4 + 12\,\frac{t_A}{L}\left(1+\frac{t_A}{L}\right). \tag{2.6}$$

The end moments and shears are then given by in terms of these coefficients :

$$M_A = -\,\bar{K}_A \frac{EI}{L}\,\theta_B - \bar{K}_c\,\frac{EI}{L}\,\theta_B, \tag{2.7}$$

$$M_B = \bar{K}_c\,\frac{EI}{L}\,\theta_A + \bar{K}_B\,\frac{EI}{L}\,\theta_B, \tag{2.8}$$

$$S_A = S_B = \frac{M_A - M_B}{L + t_A + t_B} \tag{2.9}$$

2.4 DEVELOPMENT OF EXPRESSIONS FOR MOMENT EQUILIBRIUM EQUATION FOR A JOINT AND SHEAR EQUILIBRIUM EQUATION FOR A STOREY LEVEL :

Consider the case of a single frame having m storeys and n-1 bays, subjected, at each storey level i, to lateral point loads P_i. These loads cause corresponding

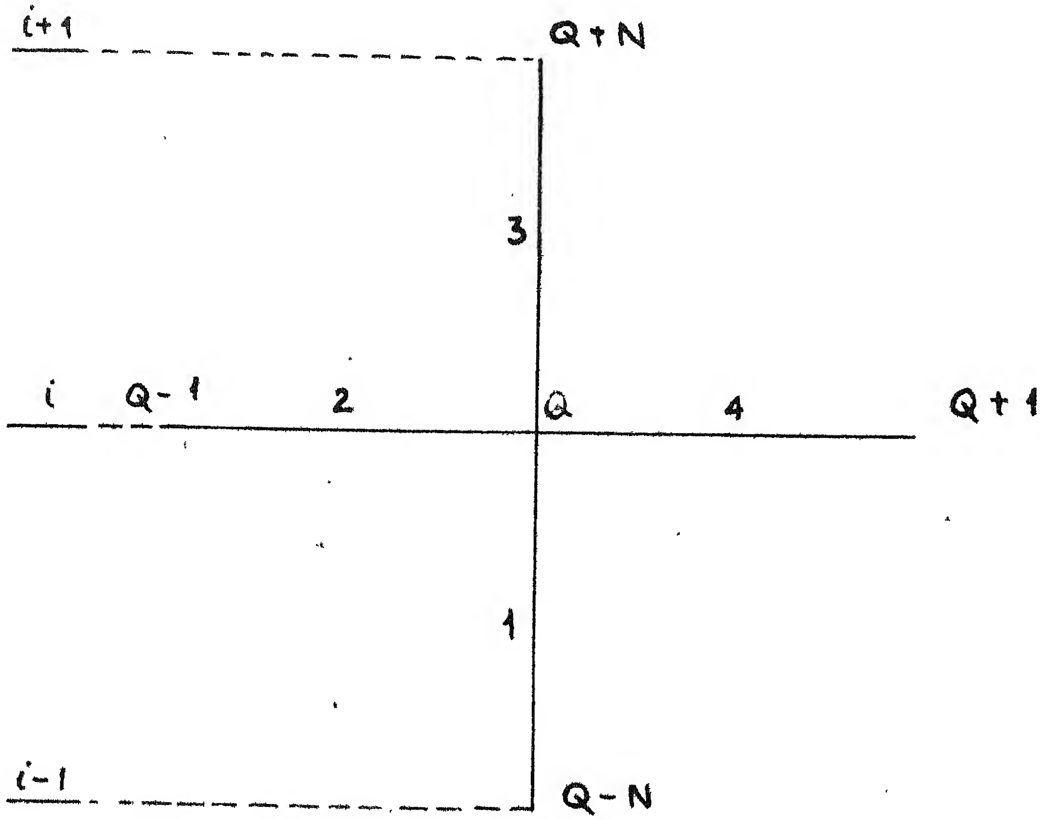


FIG. 2·3

ORIENTATION OF MEMBERS IN THE PLANE OF AN INDIVIDUAL FRAME, MEETING AT A TYPICAL JOINT.

lateral deformations Δ_i and also rotations θ_Q of each joint Q. Lateral loads and deformations are from left to right; rotations are positive clockwise. Consider the equilibrium of a joint Q at storey level i as shown in Fig. 2.3. As the joint is rigidly connected, certain relationships are immediately apparent from the compatibility of the deformations :

$$
\begin{aligned}
&(\theta_B)_1 = (\theta_B)_2 = (\theta_A)_3 = (\theta_A)_4 = \theta_q , \\
&(\theta_A)_1 = \theta_{Q-n} , \ (\theta_A)_2 = \theta_{Q-1} \\
&(\theta_B)_3 = \theta_{Q+n} , \ (\theta_B)_4 = \theta_{Q+1} , \\
&(U_B)_1 = (U_A)_3 = \Delta_i , \\
&(U_A)_1 = \Delta_{i-1} , (U_B)_3 = \Delta_{i+1}
\end{aligned}
\qquad (2.10)
$$

Since no external moments are applied to the joints of the structure, it follows that, by moment equilibrium at the joint,

$$(M_B)_1 + (M_B)_2 - (M_A)_3 - (M_A)_4 = 0 \qquad (2.11)$$

Also, from horizontal shear equilibrium, summing over all the joints at the storey level i, it follows that

$$\sum \left[(S_A)_1 - (S_B)_3 \right] = P_i \tag{2.12}$$

Substituting in equation (2.11) for the moments from equations (2.1), (2.2), (2.7), and (2.8) gives the moment equilibrium equation for joint Q as :

$$\frac{2EI_1}{L_1}\theta_{Q-n} + (\bar{K}_c)_2 \frac{EI_2}{L_2}\theta_{Q-1} + \left[4\frac{EI_1}{L_1} + (\bar{K}_B)_2 \frac{EI_2}{L_2} \right.$$

$$\left. + \frac{4EI_3}{L_3} + (\bar{K}_A)_4 \frac{EI_4}{L_4} \right] \theta_Q + (\bar{K}_c)_4 \frac{EI_4}{L_4}\theta_{Q+1} + \frac{2EI_3}{L_3}\theta_{Q+n}$$

$$+ \frac{6EI_1}{L_1^2}\Delta_{i-1} + \left[6\frac{EI_3}{L_3^2} - \frac{6EI_1}{L_1^2} \right] \Delta_i$$

$$- \frac{6EI_3}{L_3^2}\Delta_{i+1} = 0 \tag{2.13}$$

And, substituting in equation (2.12) for the shears from equations (2.3) and (2.9), gives the shear equilibrium

equation for each storey level i as :

$$\sum \left\{ - \frac{6EI_1}{L_1^2} \theta_{Q-n} + \left[\frac{6EI_3}{L_3^2} - \frac{6EI_1}{L_1^2} \right] \theta_Q + \frac{6EI_3}{L_3^2} \theta_{Q+n} \right.$$

$$\left. - \frac{12\ EI_1}{L_1^3} \Delta_{i-1} + \left[\frac{12\ EI_1}{L_1^3} + \frac{12\ EI_3}{L_3^3} \right] \Delta_i - \frac{12\ EI_3}{L_3^3} \Delta_{i+1} \right\} = P_i \tag{2.14}$$

Clearly, the set of equations, resulting from the application of equations (2.13) and (2.14) at each joint and storey level respectively in the frame, may be expressed in the matrix form :

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array} \right] \left[\begin{array}{c} \theta \\ \hline \Delta \end{array} \right] = \left[\begin{array}{c} 0 \\ \hline P \end{array} \right] \tag{2.15}$$

where A_{11} and A_{22} are square submatrices of order n and m respectively and $A_{21} = A_{12}^T$

The matrix A will be defined as

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array} \right]$$

2.5 SHEARWALL STRUCTURE TYPE I :

The shearwall structure Type I is shown in Fig. 2.4, this comprises of a shearwall with a one bay rigid frame on either side. Each storey has four degrees of freedom i.e. 3 rotations and 1 horizontal translation. The order of the square matrix A will depend on the number of storeys of the structure. The order of A will be four times the number of storeys.

A 10 - storey structure was analysed on the computer by using equations (2.13) & (2.14) and the order of the matrix A in this case is 40. The matrix A for this structure is shown in Fig. 2.5. It consists of submatrices B, B', C, D, E, F, G, H, I, J, K and L, the sizes of each of these submatrices are as follows :

$$\text{B, B}', \text{J \& K} = [2 \times 2]$$
$$\text{C, D, E, F, G \& L} = [1 \times 1]$$
$$\text{H \& I} = [1 \times 3]$$

These submatrices were arranged in the form as shown in the Fig. 2.5. Fig. 2.4 shows the numbering of members and joints of the structure.

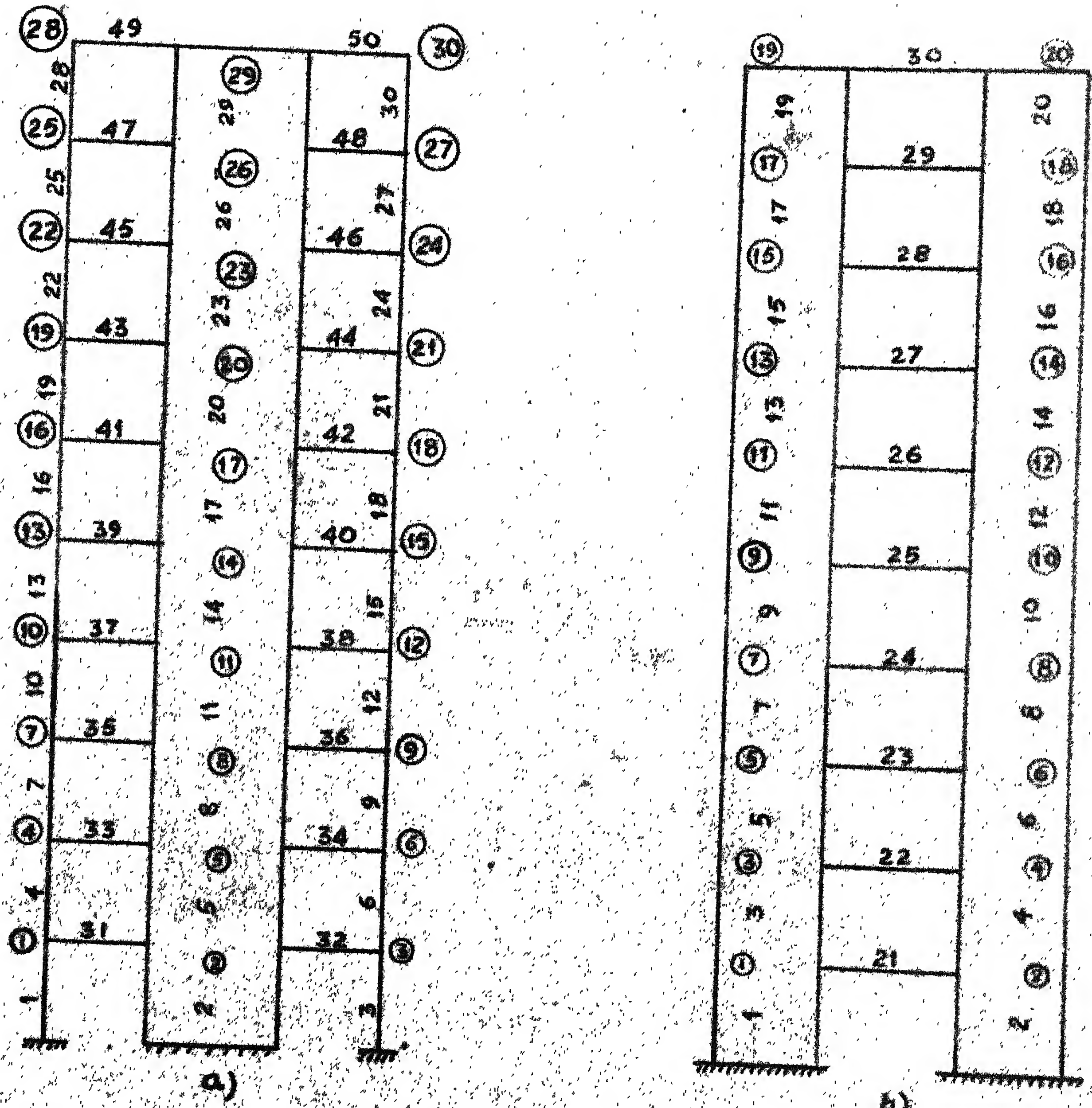


FIG. 2·4

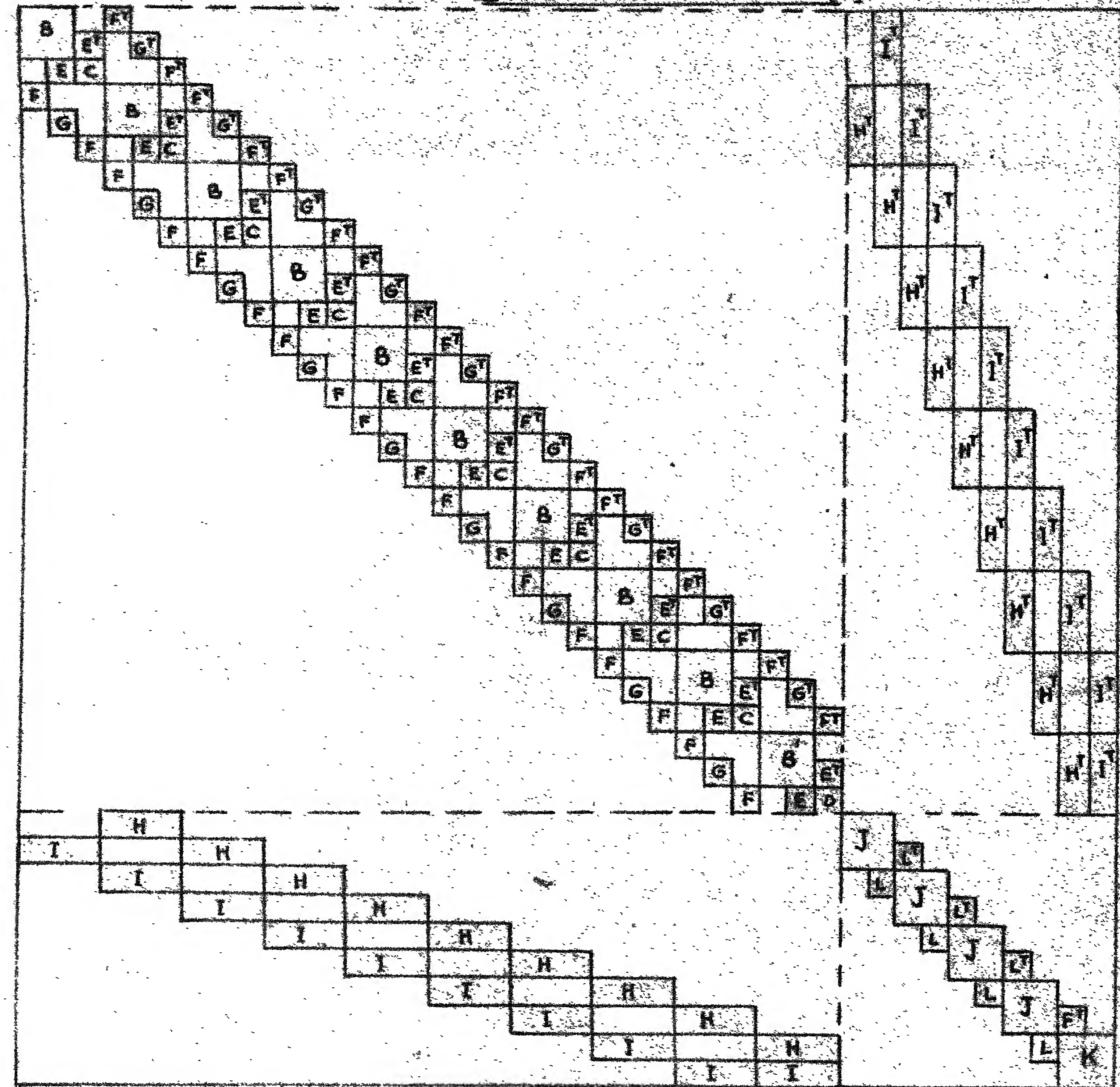
a) SHEARWALL STRUCTURE TYPE I   NUMBERING OF JOINTS AND MEMBERS

b) " " " II   NUMBERS IN CIRCLES REPRESENT JOINTS.

SHEARWALL STRUCTURE TYPE I

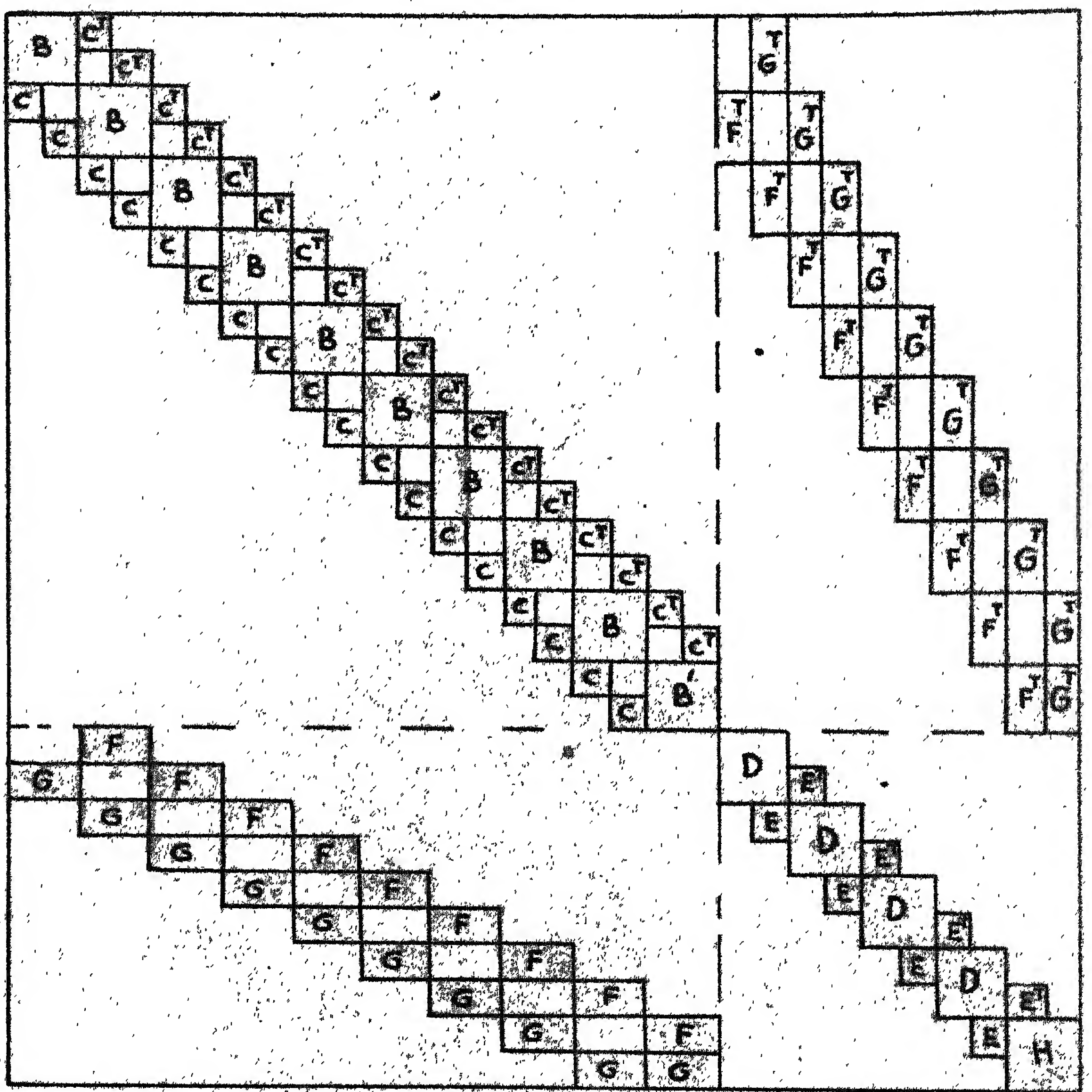
FIG. 2.5

STIFFNESS MATRIX [A]



SUBMATRICES; B, B̄, C, D, E, F, G, H, I, J, K & L

SHEARWALL STRUCTRE TYPE II



STIFFNESS MATRIX [A]

FIG. 2.6

SUBMATRICES: B, B', C, D, E, F, G & H.

CHAPTER III

DEVELOPMENT OF THE STIFFNESS MATRIX FOR SHEARWALL STRUCTURES - METHOD II

3.1 STIFFNESS METHOD :

This is also a stiffness matrix method which has certain advantages if a digital computer is available. The advantages are mainly in the relative ease of formulating the problem in this way and the simplicity of the data preparation. One important advantage of the stiffness method is that a finite element procedure can be introduced with very little further complication. This is of advantage when considering the interaction of wall and column systems with floor slabs.

The flexibility matrix, used with digital computer, is very satisfactory with moderate degrees of redundancy but becomes less satisfactory as the number of redundants increases.

Here the stiffness method has been applied to the tall buildings with shearwalls. Two types of shearwall

structures (same as in Chapter II) have been studied taking more degrees of freedom per storey.

In general, the external force R_i acting at the position and in the direction of the ith constraint will be a function of the nth joint displacements r throughout a structure having n degrees of freedom, i.e.

$$R_i = A_{i1} r_1 + A_{i2} r_2 + ---- + A_{in} rn \qquad (3.1)$$

The coefficients A_{ij} are termed "stiffness influence coefficients". The coefficients A_{ii} $(i=1,2,---,n)$ are termed "direct stiffnesses" and the A_{ij} $(i=1,2,---n,\ j \neq i)$ are termed "cross-stiffness".

A feature of the method when applied to large structural systems is that many of the cross-stiffnesses are zero and this leads to a matrix of coefficients that is "banded" in form with the nonzero elements close to the principal diagonal. The band form of the stiffness matrix leads to a substantial savings in computer time and storage.

Equation (3.1) may be written concisely for the whole structure, in matrix notation, as

$$A\ r = R \qquad (3.2)$$

where R represents a column matrix of nodal forces, r a column matrix of nodal displacements and A is the square stiffness matrix of the structure.

3.2 STIFFNESS MATRIX A FOR SHEARWALL STRUCTURE TYPE I :

Fig. 3.1 shows a structure comprised of a shearwall with a one bay rigid frame on either side. The beam-column and beam-wall joints are assumed rigid. The geometry of the structure is assumed to be regular over the whole height H. The storey height h is assumed constant and the member properties are also assumed constant.

The displacements r_i are numbered as shown in Fig. 3.1. At each floor level there are three rotations, two vertical displacements and a horizontal displacement. Axial shortening of the beams is neglected as is axial shortening of the wall in the case where the beam spans are unequal. Thus the total number of displacements is 6S where S is the number of storeys. Typical deformation and corresponding stiffnesses are shown in Fig. 3.2.

The stiffness matrix $[A]$ is formed as follows

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c} u \\ \hline \Delta \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline P \end{array}\right] \qquad (3.3)$$

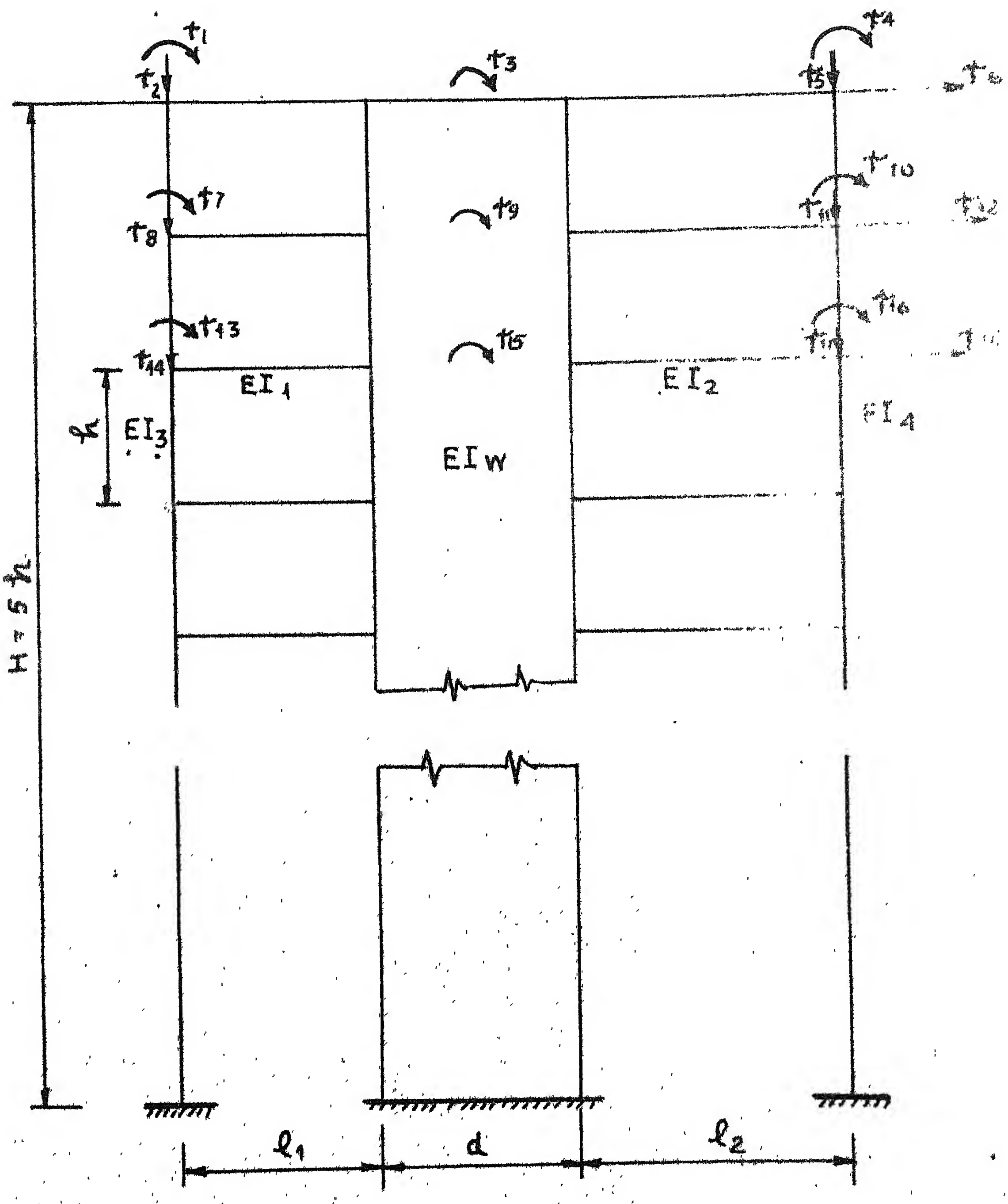


FIG 3.1

SHEARWALL STRUCTURE TYPE I

(NUMBERING OF DISPLACEMENTS)

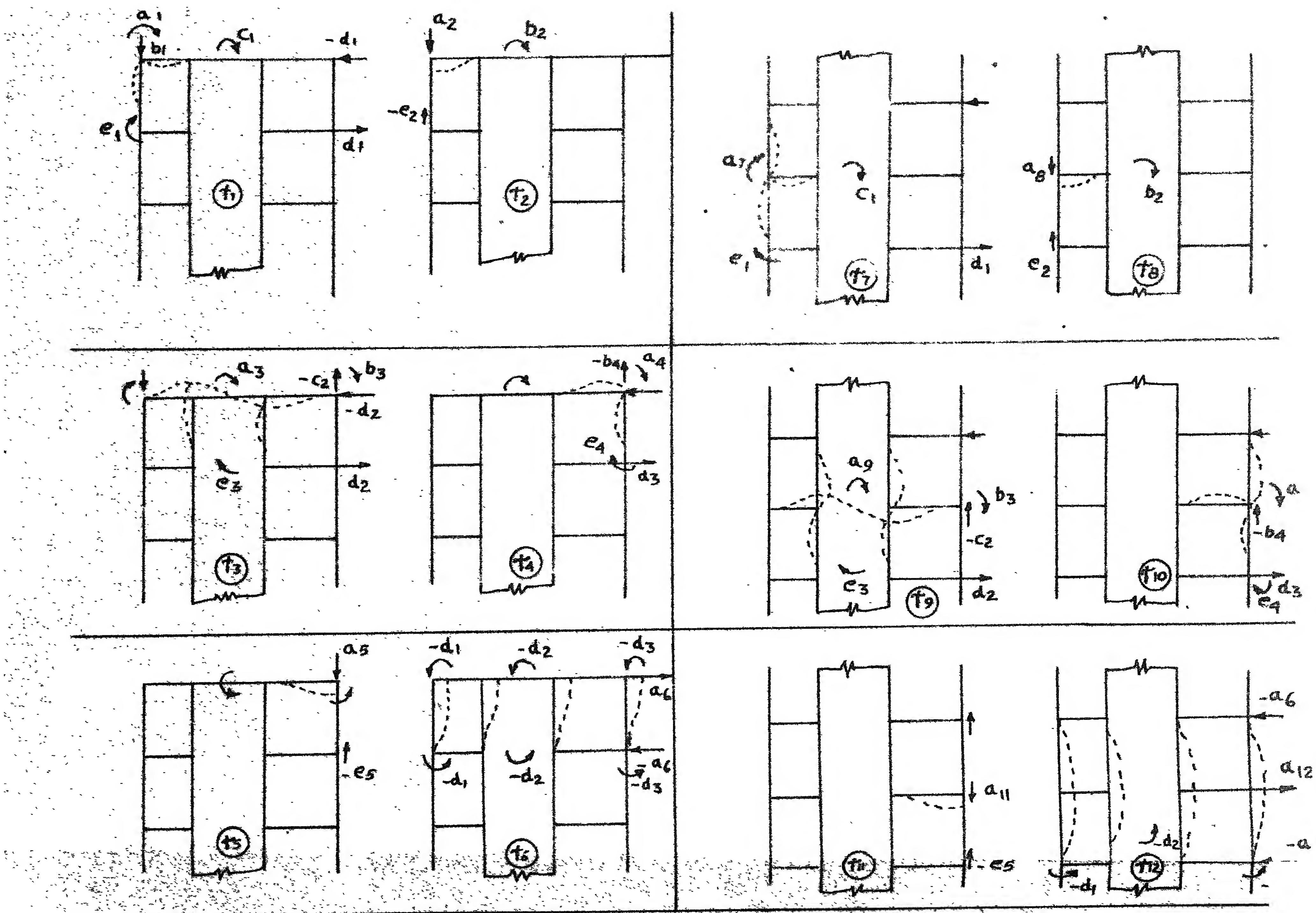


FIG 3.2

DEFORMATION AND STIFFNESSES

SHEARWALL STRUCTURE TYPE I

SHEARWALL STRUCTURE TYPE I & II

STIFFNESS MATRIX [A]

SUB-MATRICES ARE B, B′, C, D, E, F, G, H & I

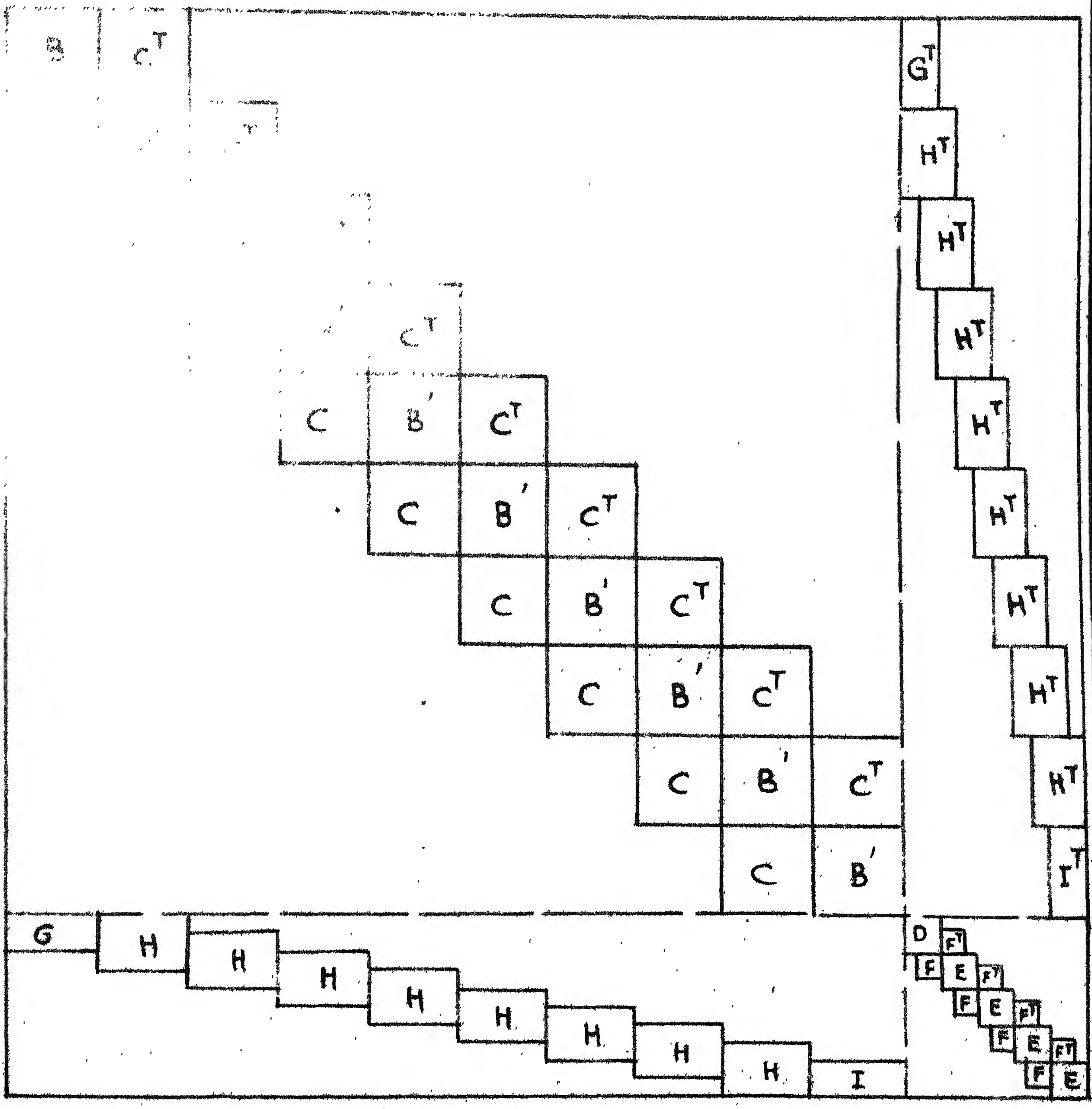


FIG. 3·3

where A_{11} and A_{22} are square submatrices of order 5S and S respectively and $A_{12} = A_{21}^T$. Δ represents the lateral horizontal displacements and u represents the displacements other than Δ i.e. rotations and axial deformations.

A 10-storey structure of this type has been analysed on the computer and the order of the matrix A is 60. The matrix for this structure is shown in the Fig. 3.3. It consists of submatrices B, B', C, D, E, F, G, H & I. The elements in these sub-matrices are as follows :

$$[B] = \begin{bmatrix} a_1 & b_1 & c_1 & 0 & 0 \\ b_1 & a_2 & b_2 & 0 & 0 \\ c_1 & b_2 & a_3 & b_3 & -c_2 \\ 0 & 0 & b_3 & a_4 & -b_4 \\ 0 & 0 & -c_2 & -b_4 & a_5 \end{bmatrix}$$

$$[B'] = \begin{bmatrix} a_7 & b_1 & c_1 & 0 & 0 \\ b_1 & a_8 & b_2 & 0 & 0 \\ c_1 & b_2 & a_9 & b_3 & -c_2 \\ 0 & 0 & b_3 & a_{10} & -b_4 \\ 0 & 0 & -c_2 & -b_4 & a_{11} \end{bmatrix}$$

$$[C] = \begin{bmatrix} e_1 & 0 & 0 & 0 & 0 \\ 0 & e_2 & 0 & 0 & 0 \\ 0 & 0 & e_3 & 0 & 0 \\ 0 & 0 & 0 & e_4 & 0 \\ 0 & 0 & 0 & 0 & e_5 \end{bmatrix}$$

$$[D] = \begin{bmatrix} a_6 & -a_6 \\ -a_6 & a_{12} \end{bmatrix}$$

$$[E] = \begin{bmatrix} a_{12} & -a_6 \\ -a_6 & a_{12} \end{bmatrix}$$

$$[F] = \begin{bmatrix} -a_6 \end{bmatrix}$$

$$[G] = \begin{bmatrix} -d_1 & 0 & -d_2 & -d_3 & 0 \\ d_1 & 0 & d_2 & d_3 & 0 \end{bmatrix}$$

TABLE 3.1 STIFFNESSES FOR SHEAR WALL STRUCTURE TYPE I

a_i	b_i	c_i	d_i	e_i
$4(\frac{k_1}{l_1} + \frac{k_3}{h})$	$\frac{6k_1}{l_1^2}$	$\frac{k_1}{l_1^2}(3d + 2l_1)$	$\frac{6k_3}{h^2}$	$\frac{2k_3}{h}$
$\frac{k_5}{h} + \frac{12k_1}{l_1^3}$	$6\frac{k_1}{l_1^3}(l_1+d)$	$\frac{6k_2(l_2+d)}{l_2^3}$	$\frac{6k_7}{h^2}$	$\frac{k_5}{h}$
$\frac{4k_7}{h} + \frac{k_1}{l_1^3}(4l_1^2 + 6dl_1 + 3d^2) + k_2/l_2^2\,(4l_2^2+6dl_2+3d^2)$	$\frac{k_2}{l_2^2}\,(2l_2+3d)$		$\frac{6k_4}{h^2}$	$\frac{2k_7}{h}$
$\frac{4k_4}{h} + \frac{4k_2}{l_2}$	$\frac{6k_2}{l_2^2}$		$\frac{a_{12}}{2} = a_6$	$\frac{2k_4}{h}$
$\frac{k_6}{h} + \frac{12k_2}{l_2^3}$				$\frac{k_6}{h}$
$\frac{12}{h^3}\,(k_3 + k_4 + k_{[illegible]})$				

i	a_i	b_i	c_i	d_i	e_i
7	$\frac{8k_3}{h} + \frac{4k_1}{l_1}$				
8	$\frac{2k_5}{h} + \frac{12k_1}{l_1^3}$				
9	$\frac{8k_7}{h} + \frac{k_1}{l_1^3}(4l_1^2 + 6dl_1 + 3d^2)$ $+ k_2/l_2^3(4l_2^2 + 6dl_2 + 3d^2)$				
10	$\frac{8k_4}{h} + \frac{4k_2}{l_2}$				
11	$\frac{2k_6}{h} + \frac{12k_2}{l_2^3}$				
12	$\frac{24}{h^3}(k_3+k_4+k_7) = 2a_6$				

k_1 = EI_1(l.h.beam)

k_2 = EI_2(r.h.beam)

k_3 = EI_3(l.h. column)

k_4 = EI_4(r.h.column)

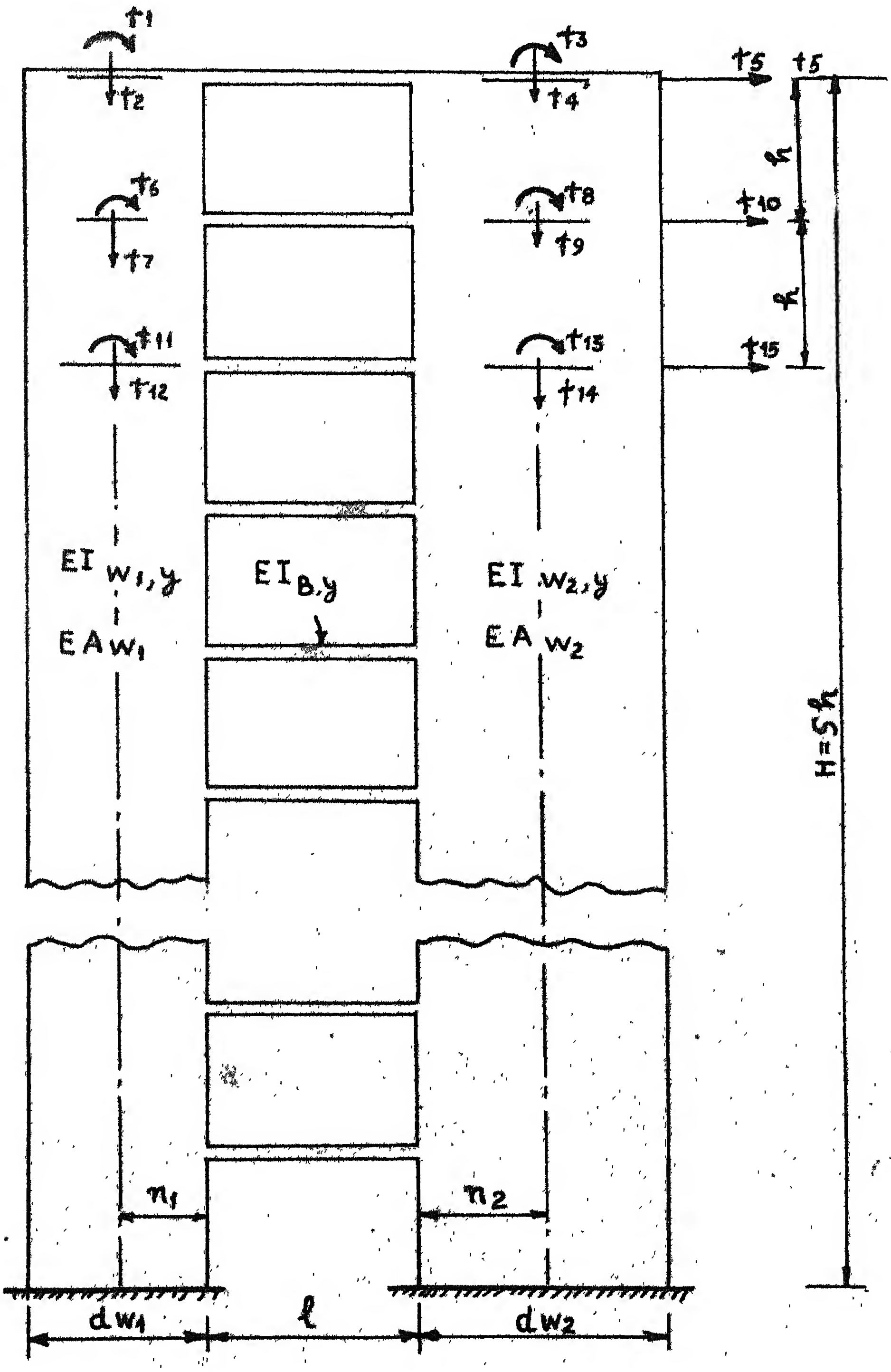
k_5 = EA_1(l.h.column)

k_6 = EA_2(r.h.column)

k_7 = EI(wall)

(NUMBERING OF DISPLACEMENTS)

FIG. 3.4.



$$H = \begin{bmatrix} -d_1 & 0 & -d_2 & -d_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ d_1 & 0 & d_2 & d_3 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} d_1 & 0 & -d_2 & -d_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

These submatrices have been arranged in the form as shown in the Fig. 3.3 by means of a computer.

The individual stiffnesses a_i, b_i, c_i, d_i and e_i are listed in the Table 3.1.

3.3 STIFFNESS MATRIX A FOR SHEARWALL STRUCTURE TYPE II :

The layout of this structure is shown in Fig. 3.4. The walls are not necessarily of the same dimensions. $EI_{W_1,y}$, $EI_{W_2,y}$ and $EI_{B,y}$ are the relevant values of flexural rigidity for walls and connecting beams. Axial deformations of the walls are included and once again it is assumed that geometry and member properties are uniform over the height of the structure.

The displacements r_i are numbered as shown in Fig. 3.4. At each floor level there are two rotations, two vertical displacements and a horizontal displacement. The total number of displacements is 5S where S is the number of storeys.

The stiffness matrix A is formed as follows :

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c} u \\ \hline \Delta \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline P \end{array}\right] \qquad (3.4)$$

where A_{11} and A_{22} are square matrices of order 4S and S respectively and $A_{12} = A_{21}^T$. Δ represents the lateral horizontal displacements and u represents the displacements other than i.e. rotations and axial deformations.

A 10-storey structure of this type has been analysed on the computer and the order of the matrix A is 50. The matrix for this structures is shown in the Fig. 3.3. It consists of submatrices B, B', C, D, E, F, G, H & I. The elements in the submatrices are as follows :

$$[B] = \begin{bmatrix} a_1 & b_1 & b_3 & -b_1 \\ b_1 & a_2 & b_2 & -b_4 \\ b_3 & b_2 & a_3 & -b_2 \\ -b_1 & -b_4 & -b_2 & a_4 \end{bmatrix}$$

$$[B'] = \begin{bmatrix} a_6 & b_1 & b_3 & -b_1 \\ b_1 & a_7 & b_2 & -b_4 \\ b_3 & b_2 & a_8 & -b_2 \\ -b_1 & -b_4 & -b_2 & a_9 \end{bmatrix}$$

$$[C] = \begin{bmatrix} b_7 & 0 & 0 & 0 \\ 0 & -b_8 & 0 & 0 \\ 0 & 0 & b_9 & 0 \\ 0 & 0 & 0 & b_{10} \end{bmatrix}$$

$$[D] = \begin{bmatrix} a_5 & -a_5 \\ -a_5 & a_{10} \end{bmatrix}$$

$$[E] = \begin{bmatrix} a_{10} & -a_5 \\ -a_5 & a_{10} \end{bmatrix}$$

$$[F] = \begin{bmatrix} -a_5 \end{bmatrix}$$

$$[G] = \begin{bmatrix} -b_6 & 0 & -b_5 & 0 \\ b_6 & 0 & b_5 & 0 \end{bmatrix}$$

$$[H] = \begin{bmatrix} b_6 & 0 & -b_5 & 0 \\ 0 & 0 & 0 & 0 \\ b_6 & 0 & b_5 & 0 \end{bmatrix}$$

$$[I] = \begin{bmatrix} -b_6 & 0 & -b_5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

These submatrices have been arranged in the matrix A as shown in the Fig. 3.3 by means of a computer.

The individual stiffnesses a_i and b_i are listed in the Table 3.2.

BLE 3.2 STIFFNESSES FOR SHEAR WALL STRUCTURE TYPE II

Stiffness Coefficients

i	a_i	b_i
1	$\frac{2k_1}{h}\left(\frac{2+g_1}{1+2g_1}\right)+\frac{4k_3}{l^3}\left(l^2+3n_1l+3n_1^2\right)$	$\frac{6k_3}{l^3}\left(1+2n_1\right)$
2	$\frac{k_4}{h}+\frac{12k_3}{l^3}$	$\frac{6k_3}{l_3}\left(1+2n_2\right)$
3	$\frac{2k_2}{h}\left(\frac{2+g_2}{1+2g_2}\right)+\frac{4k_3}{l^3}\left(l^2+3n_2l+3n_2^2\right)$	$\frac{2k_3}{l^3}\left(l^2+3n_1l+3n_2l+6n_1n_2\right)$
4	$\frac{k_5}{h}+\frac{12k_3}{l^3}$	$\frac{12k_3}{l^3}$
5	$\frac{12}{h^3}\left[\frac{k_1}{(1+2g_1)}+\frac{k_2}{(1+2g_2)}\right]$	$\frac{6k_2}{h^2}\left(\frac{1}{1+2g_2}\right)$
6	$\frac{4k_1}{h}\left(\frac{2+g_1}{1+2g_1}\right)+\frac{4k_3}{l^3}\left(l^2+3n_1l+3n_1^2\right)$	$\frac{6k_1}{h^2}\left(\frac{1}{1+2g_1}\right)$

i	a_i	b_i
7	$\frac{2k_4}{h} + \frac{12k_3}{l^3}$	$\frac{2k_1}{h}\left(\frac{1-g_1}{1+2g_1}\right)$
8	$\frac{4k_2}{h}\left(\frac{2+g_2}{1+2g_2}\right) + \frac{4k_3}{l^3}(l^2+3n_2l+3n_2^2)$	$\frac{k_4}{h}$
9	$\frac{2k_5}{h} + \frac{12k_3}{l^3}$	$\frac{2k_2}{h}\left(\frac{1-g_2}{1+2g_2}\right)$
10	$2a_5$	$\frac{k_5}{h}$

Data $k_1 = EI_{w1,y}$ n_1 f = shape factor

$k_2 = EI_{w2,y}$ n_2 $g_1 = \frac{6fEI_{w1,y}}{GA_{w1}h^2}$

$k_3 = EI_{B,y}$ l Fig.3.4 $g_2 = \frac{6fEI_{w2,y}}{GA_{w2}h^2}$

$k_4 = EA_{w1}$ h

$k_5 = EA_{w2}$

CHAPTER IV

FORMULATION OF LATERAL STIFFNESS MATRIX

The matrix A is partitioned into A_{11}, A_{12}, A_{21} and A_{22} as follows :

$$[A] = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \quad , \text{ so that the}$$

horizontal translations and displacements other than horizontal translations e.g. rotations and axial deformations, are separated.

The equation relating displacements and forces can be written in the form

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c} u \\ \hline \Delta \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline P \end{array}\right] \tag{4.1}$$

where the submatrices of matrix A have already been defined. P represents horizontal forces at floor levels. Δ represents the horizontal translations and u denotes

the displacements other than Δ

If the matrix F is defined as $F = A^{-1}$, it follows that

$$\left[\begin{array}{c|c} F_{11} & F_{12} \\ \hline F_{21} & F_{22} \end{array}\right] \left[\begin{array}{c} 0 \\ \hline P \end{array}\right] = \left[\begin{array}{c} u \\ \hline \Delta \end{array}\right] \tag{4.2}$$

The submatrices of F being of the same order as the corresponding submatrices of A.

Equation (4.2) yields

$$F_{22} \; P = \Delta$$

or
$$P = F_{22}^{-1} \Delta \tag{4.3}$$

This equation expresses a direct relationship between the applied loads and the corresponding displacements.

The matrix F_{22}^{-1} may, however, be expressed directly in terms of the submatrices of A as is demonstrated below.

Since $F = A^{-1}$, it follows that

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c|c} F_{11} & F_{12} \\ \hline F_{21} & F_{22} \end{array}\right] = \left[\begin{array}{c|c} I_1 & 0_1 \\ \hline 0_2 & I_2 \end{array}\right] \tag{4.4}$$

Multiplying the partitioned matrices

$$A_{11}\ F_{12} + A_{12}\ F_{22} = 0_1 \tag{4.5}$$

Similarly

$$A_{21}\ F_{12} + A_{22}\ F_{22} = I_2 \tag{4.6}$$

Premultiplying equation (4.5) by A_{11}^{-1}, we get

$$F_{12} + A_{11}^{-1}\ A_{12}\ F_{22} = 0_1$$

or
$$F_{12} = -\ A_{11}^{-1}\ A_{12}\ F_{22} \tag{4.7}$$

Substituting the value of F_{12} in equation (4.6) we obtain

$$-A_{21}\ A_{11}^{-1}\ A_{12}\ F_{22} + A_{22}\ F_{22} = I_2$$

$$F_{22} = {A_{22} - A_{21}\ A_{11}^{-1}\ A_{12}}^{-1}$$

Taking inverse of both sides

$$F_{22}^{-1} = A_{22} - A_{21}\ A_{11}^{-1}\ A_{12} \tag{4.8}$$

Thus, for a structure, the lateral displacement may be determined directly from the lateral loads by solution of the following matrix equation :

$$K \Delta = P \tag{4.9}$$

where

$$K = A_{22} - A_{21} A_{11}^{-1} A_{12}$$

The computer programmes were written which generated the matrices A and finally gave the K matrix for direct use in dynamic analysis.

SHEARWALL STRUCTURE TYPE I & II

ELEMENTS OF LATERAL STIFFNESS MATRIX [K]

	1	2	3	4	5	6	7	8	9	10
1	K_{11}									
2	K_{21}	K_{22}								
3	K_{31}	K_{32}	K_{33}				SYMMETRIC			
	K_{41}	K_{42}	K_{43}	K_{44}						
	K_{51}	K_{52}	K_{53}	K_{54}	K_{55}					
			K_{63}	K_{64}	K_{65}	K_{66}				
			K_{73}	K_{74}	K_{75}	K_{76}	K_{77}			
			K_{83}	K_{84}	K_{85}	K_{86}	K_{87}	K_{88}		
			K_{93}	K_{94}	K_{95}	K_{96}	K_{97}	K_{98}	K_{99}	
			$K_{10,3}$	$K_{10,4}$	$K_{10,5}$	$K_{10,6}$	$K_{10,7}$	$K_{10,8}$	$K_{10,9}$	$K_{10,10}$

FIG. 4·1

CHAPTER - V

DETERMINATION OF NATURAL FREQUENCIES AND MODE SHAPES

The equation of motion of a freely vibrating, undamped system[11] can be written in the matrix form as

$$m \; (\ddot{x}) + [K] \; (x) = 0 \qquad (5.1)$$

where

$\lceil m \rfloor$ = a diagonal matrix containing masses of the system

$[K]$ = square stiffness matrix

$(\ddot{x}), (x)$ = column matrices of accelerations and displacements, respectively.

5.1 THE DYNAMICAL MATRIX $[D]$[10]

The above system can also be expressed in the form of the matrix $[D]$, which is called the dynamical matrix. This matrix is defined by

$$[D] = [K]^{-1} \; \lceil m \rfloor = [\Phi] \; \lceil m \rfloor \qquad (5.2)$$

where

$$[\Phi] = [K]^{-1}$$

If the matrix differential equation (5.1) is premultiplied by $[K]^{-1}$, the result may be written in the form

$$[D]\,(\ddot{x}) + (x) = 0 \qquad (5.3)$$

5.2 THE INVERSE DYNAMICAL MATRIX [W]

If matrix $[K]$ is a singular matrix and, therefore, $[K]^{-1}$ does not exist. In this case the dynamical matrix $[D]$ cannot be obtained. In such case it is convenient to introduce the inverse dynamical matrix $[W] = [D]^{-1}$. This matrix is given by

$$[W] = [m]^{-1}\,[K] = [D]^{-1} \qquad (5.4)$$

= Inverse Dynamical Matrix.

To obtain the equations of motion of the system in terms of the inverse dynamical matrix, it is necessary only to premultiply equation (5.1) by $[m]^{-1}$ and obtain

$$(\ddot{x}) + [W](x) = 0 \tag{5.5}$$

It is therefore, seen that the differential equations of the system may be expressed in the equivalent forms (5.3) and (5.5). In general, the matrices $[D]$ and $[W]$ are not symmetric matrices.

5.3 THE NATURAL FREQUENCIES AND MODES[9,10]

In order to study the oscillations of the general system formulated in terms of the inverse dynamical matrix $[W]$ by (5.5), let us search for an oscillatory solution of this matrix differential equation of the form

$$(x) = (\bar{x}) \, \mathrm{Sin} \, (\omega t + \theta) \tag{5.6}$$

where $(\bar{x})$ is a column matrix of n unknown amplitudes, is an angular frequency to be determined, and θ is a phase angle. If this assumed solution is substituted into (5.5), the following result is obtained:

$$-\omega^2 (\bar{x}) + [W](\bar{x}) = (0) \tag{5.7}$$

after the trigonometric function Sin $(\omega t + \theta)$ has been divided from both terms. If we let

$$\omega^2 = \mu, \tag{5.8}$$

the equation (5.7) may be written in the following alternate form:

$$(\mu U - [W])(\bar{x}) = ((0)) \tag{5.9}$$

where

U = nth order unit matrix

This equation represents a set of linear homogeneous equations in the elements of the column matrix $(\bar{x})$. In order for a nontrivial solution of these equations to be permissible, it is necessary that the determinent of the coefficients of the elements of $(\bar{x})$ vanish. We have, therefore,

$$\det(\mu U - [W]) = 0 \tag{5.10}$$

This equation is recognised as the characteristic equation of the inverse dynamical matrix $[W]$. In general, it is an equation of the nth degree. Let it be assumed the n roots of equation (5.10) are distinct and have the values, $\mu_1, \mu_2 \ldots, \mu_n$. These roots are the eigenvalues of $[W]$. To each

eigenvalue μ_i, there corresponds a natural frequency of oscillation ω_i and a time period T_i given in the form

$$\omega_i = (\mu_i)^{1/2} \tag{5.11}$$

and

$$T_i = \frac{2\pi}{\omega_i} \tag{5.12}$$

where $i = 1, 2, 3, ----, n.$

The inverse dynamical matrix $[W]$ was computed and frequencies, time periods and mode shapes were computed by using a standard subroutine referred at (9).

5.4 THE MODAL COLUMNS

From equation (5.9) it can be seen that to each eigenvalue μ_i there corresponds a column $(\bar{x})_i$ or eigenvector of the matrix $[W]$ that satisfies the equation

$$[W]\,(\bar{x})_i = \mu_i\,(\bar{x})_i\ , \quad i = 1,2,3, ---, n. \tag{5.13}$$

In the literature of vibration theory, the eigenvectors $(\bar{x})_i$ are called modal columns.

5.5 ORTHOGONALITY OF THE MODAL COLUMNS

Since $[W] = [m]^{-1}[K]$, equation (5.13) may be also be written in the form

$$[K](\bar{x})_i = \mu_i [m](\bar{x})_i \tag{5.14}$$

Let us write the same equation with subscript j, so that

$$[K](\bar{x})_j = \mu_j [m](\bar{x})_j \tag{5.15}$$

If we now premultiply (5.14) by $(\bar{x})_j^T$ and (5.15) by $(\bar{x})_i^T$, we obtain

$$(\bar{x})_j^T [K] (\bar{x})_i = \mu_i (\bar{x})_j^T [m] (\bar{x})_i \tag{5.16}$$

$$(\bar{x})_i^T [K] (\bar{x})_j = \mu_j (\bar{x})_i^T [m] (\bar{x})_j \tag{5.17}$$

We know that $[K]$ and $[m]$ are symmetric matrices; hence, if we take the transpose of both sides of equation

(5.17), we obtain

$$(\bar{x})_j^T \quad [K] \quad (\bar{x})_i = \mu_j \, (\bar{x})_j^T \quad [m] \, (\bar{x})_i \qquad (5.18)$$

If we now subtract equation (5.18) from (5.16), the result is

$$0 = (\mu_i - \mu_j) \, (x)_j^T \quad [m] \quad (\bar{x})_i \qquad (5.19)$$

Since, by hypothesis, u_i and u_j are two distinct eigenvalues, we must have

$$(\bar{x})_j^T \quad [m] \quad (\bar{x})_i = 0 \qquad i \neq j \qquad (5.20)$$

If this is now substituted into equation (5.18) we see that we must also have

$$(\bar{x})_j^T \quad [K] \, (\bar{x})_i = 0 \qquad i \neq j \qquad (5.21)$$

The relations (5.20) and (5.21) are a form of generalised orthogonal relations satisfied by the modal columns or eigenvectors of the inverse dynamical matrix $[W]$.

5.6 THE MODAL MATRIX $\bar{x}$

We now construct a partitioned square matrix $[\bar{x}]$ by placing the modal columns or eigenvectors $(\bar{x})_i$ side by side to form a square array of numbers in the form

$$[\bar{x}] = \left[(\bar{x})_1, \ (\bar{x})_2, \ - - - - , \ (\bar{x})_n\right]$$

$$= \left[x_{ji}\right] \tag{5.22}$$

This square matrix $[\bar{x}]$ is called the modal matrix of the matrix $[W]$.

The set of equations (5.13) may be written as a single equation in terms of the modal matrix. This equation has the form

$$[W][\bar{x}] = [\bar{x}]\ [d] \tag{5.23}$$

where $[d]$ is a diagonal matrix whose elements are the eigenvalues μ_i. $[d]$ is called the spectral matrix and has the form

$$[d] = \begin{bmatrix} \mu_1 & & & & 0 \\ & \mu_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ 0 & & & & \mu_n \end{bmatrix} \tag{5.24}$$

If the eigenvalues of $[W]$ are distinct, $[\bar{x}]$ can be shown to be nonsingular and, therefore, to have an inverse $[\bar{x}]^{-1}$. If the equation (5.23) is post-multiplied by $[\bar{x}]^{-1}$, the result is

$$[W] = [\bar{x}] \; [d] \; [\bar{x}]^{-1} \tag{5.25}$$

Therefore, the modal matrix $[\bar{x}]$ reduces the inverse dynamical matrix $[W]$ to the diagonal form.

5.7 THE EIGENVECTORS OF THE DYNAMICAL MATRIX $[D]$

The equation (5.13) indicates that the column $(\bar{x})_i$ is an eigenvector of the inverse dynamical matrix $[W]$ associated with the eigenvalue μ_i. Now let us determine the relations between the eigenvectors of $[W]$ and the eigenvectors of the dynamical matrix $[D]$. In order to discover these relations, we may write (5.13) inthe form

$$[D]^{-1} \; (\bar{x})_i = \mu_i \; (\bar{x})_i \tag{5.26}$$

Or, if (5.26) is premultiplied by $[D]$, the result may be written in the form

$$[D](\bar{x})_i = \frac{1}{\mu_i}(\bar{x})_i = Z_i(\bar{x})_i \qquad (5.27)$$

where $Z_i = \frac{1}{\mu_i}$. It is, therefore, evident from (5.27) that the column $(\bar{x})_i$ is also an eigenvector of the dynamical matrix $[D]$ but is associated with the eigenvalue $\frac{1}{\mu_i} = Z_i$ of the matrix $[D]$. The angular frequency ω_i of the system corresponding the eigenvalue μ_i is given by (5.11) in the form

$$\omega_i = (\mu_i)^{1/2} = \left(\frac{1}{Z_i}\right)^{1/2}, \quad i = 1, 2, 3, ---, n \qquad (5.28)$$

We see, therefore, that the eigenvector $(\bar{x})_i$ is the modal column associated with the angular frequency ω_i regardless of whether the analysis is formulated in terms of the inverse dynamical matrix $[W]$ or the dynamical matrix $[D]$.. It is seen from (5.28) that the largest eigenvalue of $[D]$ gives the lowest frequency of oscillation, whereas the largest eigenvalue of $[W]$ gives the highest frequency of the system.

CHAPTER VI

MODAL ANALYSIS OF MULTIDEGREE SYSTEMS AND ITS APPLICATION TO WIND AND EARTHQUAKE ANALYSIS

6.1 MODAL ANALYSIS :

The response of multidegree elastic systems due to applied forces or initial conditions is determined by the modal method, in which the responses in the normal modes are determined separately, and then superimposed to provide the total response. Each normal mode is treated as an independent single-degree system.

The applicability of the modal method of analysis is limited to linearly elastic systems and to cases in which all forces applied to the structure have the same time variation. Also the damping has been neglected.

The equations of motion for a multidegree lumped-mass system may be written as

$$[m]\ (x) + [K]\ (x) = (F(t)) \qquad (6.1)$$

where $[m]$, $[K]$, $(\ddot{x})$ and (x) have the usual meaning as per equation (5.1) and

$(F(t))$ = a column matrix of applied dynamic forces.

For linear structural systems, $[K]$ is symmetric.

The equation (6.1) can also be written as

$$m_i \ddot{x}_i + \sum_{j=1}^{n} K_{ij} x_j = f_i (t) \qquad (6.2)$$

$$(i = 1,2,\text{----}., n)$$

Let the solutions of equations (6.2) be taken in the form

$$x_i(t) = \sum_{k=1}^{n} \bar{x}_i^k \; T_k (t) \; ; \; i = 1,2,\text{----}., n. \qquad (6.3)$$

Here T_k (t) are called the "Normal Co-ordinates" and these are to be found. $\bar{x}_i^k$ $(k = 1,2,\text{----},n)$ form the mode shapes.

Substituting equation (6.3) into (6.2) we get

$$m_i \sum_{k=1}^{n} \bar{x}_i^k \ddot{T}_k + \sum_{j=1}^{n} K_{ij} \left(\sum_{k=1}^{n} \bar{x}_j^k T_k\right) = f_i(t) \qquad (6.4)$$

This equation may be written with the interchange of summation signs;

$$m_i \sum_{k=1}^{n} \bar{x}_i^k \ddot{T}_k + \sum_{k=1}^{n} \left(\sum_{j=1}^{n} K_{ij} \bar{x}_j^k \right) T_k = f_i(t) \qquad (6.5)$$

The equation (5.14) may be written as follows :

$$\sum_{j=1}^{n} K_{ij} \bar{x}_j^k = m_i \omega_k^2 \bar{x}_i^k \qquad (6.6)$$

Substituting equation (6.6) into (6.5) we obtain

$$m_i \sum_{k=1}^{n} \bar{x}_i^k \ddot{T}_k + \omega_k^2 T_k = f_i (t) \qquad (6.7)$$

If we multiply this equation by $\bar{x}_i^l$ and sum over 'i', due to orthogonal property, all terms in the left side of above equation will be zero except the term for which $\omega_k = \omega_l$ and we get

$$\ddot{T}_k + \omega_k^2 T_k = \frac{\sum_{i=1}^{n} \bar{x}^k f_i (t)}{\sum_{i=1}^{n} m_i \bar{x}_i^{k^2}} = F_k (t) \qquad (6.8)$$

The equation (6.8) is similar to the differential equation of single degree of freedom system and term $F_k(t)$ is called 'Generalised Force'. The solution of equation (6.8) is

$$T_k = A_k \cos \omega_k t + B_k \sin \omega_k t + \frac{1}{\omega_k} \int_0^t F_k(\tau) \sin \omega_k (t-\tau)\, d\tau \qquad (6.9)$$

Therefore, substituting the value of T_k in equation (6.3),

$$x_i = \sum_{k=1}^{n} \bar{x}_i^k \Big[A_k \cos \omega_k t + B_k \sin \omega_k t + \frac{1}{\omega_k} \int_0^t F_k(\tau) \sin \omega_k (t-\tau)\, d\tau \Big] \quad i = 1,2,\text{---}n. \qquad (6.10)$$

In the above equations, the constants A_k and B_k are determined by initial conditions as

$$\begin{aligned} x_i(0) &= \sum_{k=1}^{n} \bar{x}_i^k A_k = x_i^0 \\ \dot{x}_i(0) &= \sum_{k=1}^{n} \bar{x}_i^k \omega_k B_k = \dot{x}_i^0 \end{aligned} \qquad (6.11)$$

Using the orthogonality property we can determine the constants as

$$A_k = \frac{\sum_{i=1}^{n} m_i \, \bar{x}_i^k \, x_i^o}{\sum_{i=1}^{n} m_i \, \bar{x}_i^{k^2}}$$

$$B_k = \frac{\sum_{i=1}^{n} m_i \, \bar{x}_i^k \, \dot{x}_i^o}{\omega_k \sum_{i=1}^{n} m_i \, \bar{x}_i^{k^2}} \tag{6.12}$$

This completes the solution of discrete system with lumped masses.

The shear force at any mass position 'i' is given by

$$(S.F)_i = \sum_{j=1}^{n} K_{ij} \, x_j \tag{6.13}$$

If the external load on the system is in the form

$f_i(t) = F_i \Phi(t)$, then

$$F_k(t) = \frac{\sum_{i=1}^{n} \bar{x}_i^k F_i}{\sum_{i=1}^{n} m_i \bar{x}_i^{k^2}} \phi(t) \qquad (6.14)$$

and

$$T_k(t) = \frac{\sum_{i=1}^{n} \bar{x}^k F_i}{\omega_k^2 \sum_{i=1}^{n} m_i \bar{x}^{k^2}} \int_0^t \omega \phi(\tau) \sin\omega_k (t-\tau)\, d\tau$$

$$= C_k \,(DLF)_k \qquad (6.15)$$

where

C_k = Modal Static Deflection

and $(DLF)_k$ = Dynamic Load Factor

Substituting the value of $T_k(t)$ in equation 6.3, we get

$$x_i(t) = \sum_{k=1}^{n} C_k \bar{x}_i^k (DLF)_k \qquad (6.16)$$

$$(i=1,2,\text{---},n)$$

Maximum value of the displacement at any position 'i' can be calculated by using the maximum value of DLF corresponding to frequency ω_k or period $T_k = \frac{2\pi}{\omega_k}$ from the force spectral curves. We cannot sum the modal maximums over K as in equation (6.16) as we do not know the phase difference. All the modal maximums may not occur at the same instance of time. To overcome this we can have two estimates, one of which is upper bound and the other is lower bound.

$$x_i\,(\max) = \sum_{k=1}^{n} C_k\,\bar{x}_i^k\,(DLF)_{k,\,\max} \qquad \text{(upper bound)} \quad (6.17)$$

$$x_i,(\max) = \sqrt{\sum_{k=1}^{n} \left\{C_k\,\bar{x}_i^k\,(DLF)_{k,\,\max}\right\}^2} \qquad \text{(lower bound)}$$

Similarly the upper and lower estimates of shear force (equation 6.13) are found as follows

$$(S.F)_i\max = \sum_{k=1}^{n} \left|(S.F)_k\right| \qquad \text{(upper bound)} \quad (6.19)$$

$$(S.F)_i\max = \sqrt{\sum_{k=1}^{n} \left\{(S.F.)_k\right\}^2} \qquad \text{(lower bound)} \quad (6.20)$$

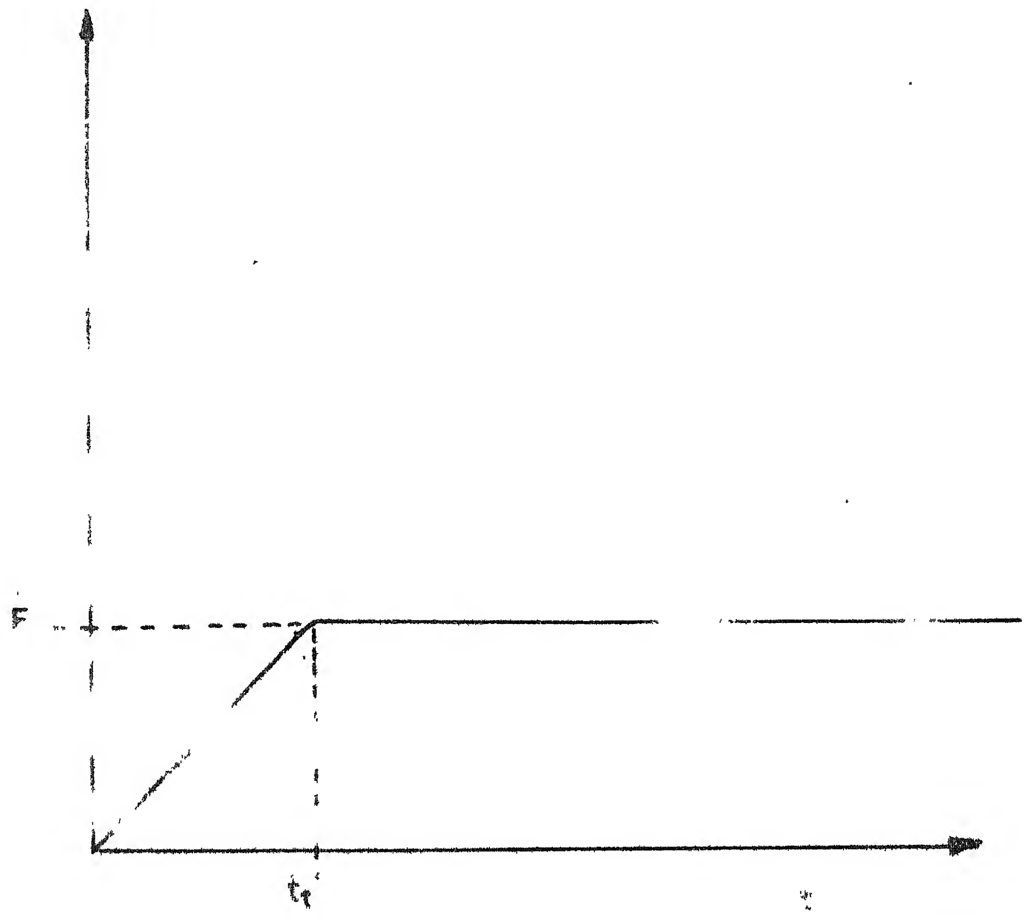


FIG 6·1

INDEALISED WIND LOADING

t_r = RISE TIME
$f(t)$ = WIND FORCE

then evaluating the integrals in equations (6.22) we obtain

$$DLF = \left(\frac{t/T}{t_r/T} - \frac{\sin\frac{2\pi t}{T}}{\frac{2\pi t}{T}}\right) \quad \text{for } t \leq t_r$$

$$DLF = \left(1 - \frac{\sin\frac{2\pi t}{T}}{\frac{2\pi t_r}{T}} + \frac{\sin 2\pi\left(\frac{t}{T} - \frac{t_r}{T}\right)}{\frac{2\pi t_r}{T}}\right) \quad \text{for } t \geq t_r \tag{6.23}$$

The plot of the maximum values of DLF for various values of $\frac{t}{T}$ is shown in Fig. A.1.

Applying equation (6.16), we can obtain the values of displacement and their upper and lower bounds by equations (6.17) and (6.18) respectively. Similarly the upper and lower estimates of shear force are found by using equations (6.19) and (6.20) respectively.

6.3 APPLICATION TO EARTHQUAKE ANALYSIS :

Consider the discrete structure subjected to base vibration (Fig. 6.2) during earthquake. x_i is the relative lateral displacement of ith mass m_i relative to ground. y is the ground displacement. Then the mass at station i is

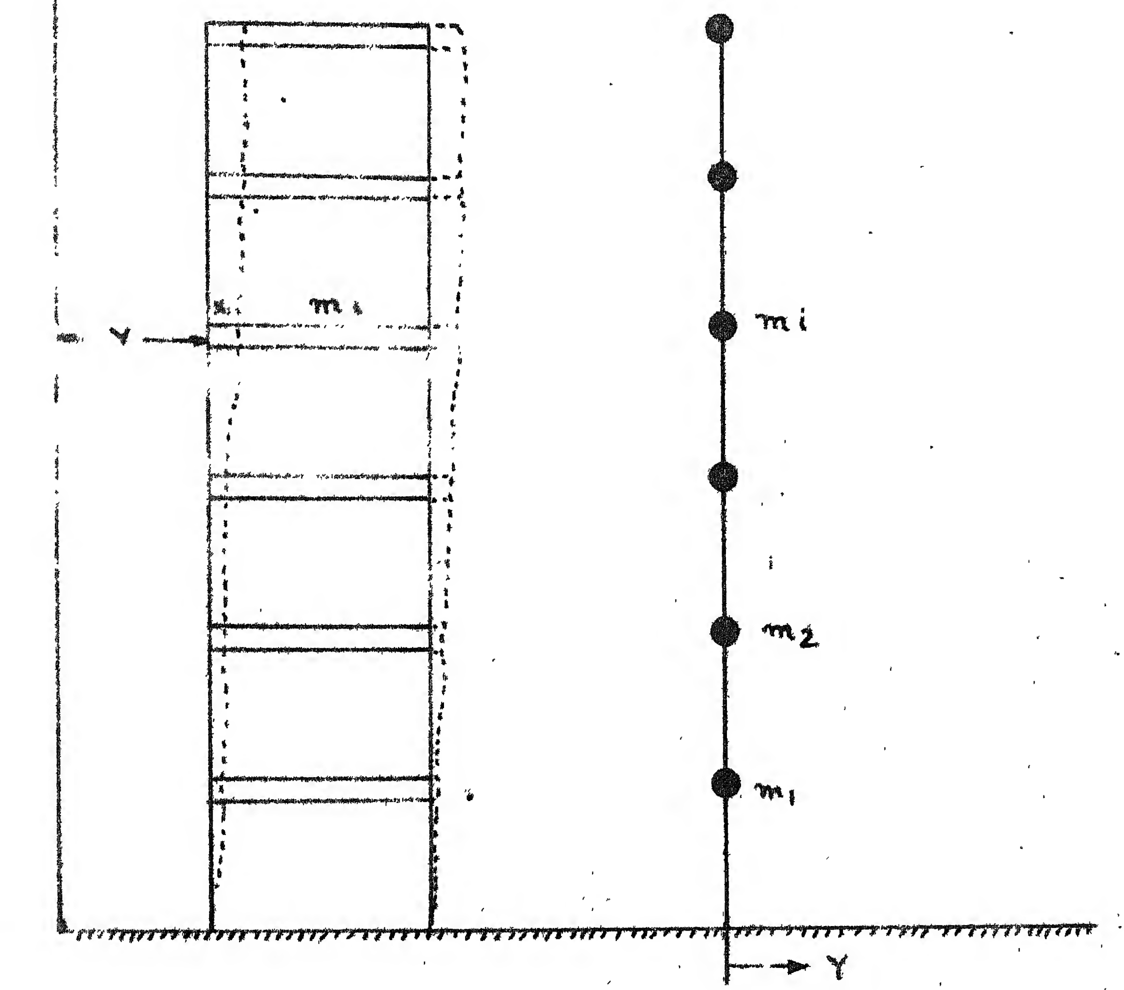


FIG. 6:2

STRUCTURE SUBJECTED TO BASE MOTION DURING EARTHQUAKE

subjected to an absolute acceleration of $(\ddot{y} + \ddot{x}_i)$. Then the equation (6.2) without the external force $f_i(t)$ takes the form

$$m_i (\ddot{x}_i + \ddot{y}) + \sum_{j=1}^{n} K_{ij} x_j = 0 \qquad (6.24)$$

or

$$m_i \ddot{x}_i + \sum_{j=1}^{n} K_{ij} x_j = - m_i \ddot{y} \qquad (6.25)$$

$$(i = 1, 2, ---, n)$$

Solutions of these equations will be

$$x_i = \sum_{k=1}^{n} \bar{x}_i^k \, T_k (t)$$

where

$$T_k(t) = \frac{-\sum_{i=1}^{n} \bar{x}_i^k m_i}{\dot{k} \sum_{i=1}^{n} \bar{x}_i^{k^2} m_i} \int_0^t \phi (\tau) \sin \omega_k (t-\tau) \, d\tau \qquad (6.26)$$

Now

$$T_{k,\ max} = (MPF)_k \quad S_{vk}$$

where

$$(MPF)_k = \frac{\sum_{i=1}^{n} m_i \bar{x}_i^k}{\omega_k \sum_{i=1}^{n} m_i \bar{x}_i^{k^2}} \tag{6.27}$$

= Modal Participation Factor

S_{vk} = Value from velocity spectrum for ω_k

$$= - \int_0^t \Phi(\tau) \sin \omega_k (t-\tau)\, d\tau \text{ for undamped system.}$$

The expression for S_v for damped system is as follows

$$S_v = \left[- \int_0^t y(\tau)\, \bar{e}^{\omega \zeta (t-\tau)} \cos \omega_d (t-\tau)\, d\tau + \frac{\zeta^2}{\sqrt{1-\zeta^2}} \int_0^t y(\tau)\, \bar{e}^{\omega \zeta (t-\tau)} \sin \omega_d (t-\tau)\, d\tau \right]_{max} \tag{6.28}$$

where

ω = Undamped natural frequency of the system,

$\omega = \frac{2\pi}{T}$ where T is the undamped natural period

ω_d = Damped natural frequency of vibration,

ξ = the fraction of critical damping, &

$\dot{y}$ = the ground displacement.

S_v is plotted for different values of ξ and T, from where we can get its value.

Then

$$x_{i,\, max} = \sum_{k=1}^{n} \left| (MPF)_k \; \bar{x}_i^k \; S_{vk} \right| \text{ (upper bound)} \qquad (6.29)$$

and

$$x_{i,max} = \sqrt{\sum_{k=1}^{n} \left\{ (MPF)_k, \; \bar{x}_i^k \; S_{vk} \right\}^2} \text{ (lower bound)} \qquad (6.30)$$

Maximum shear at any mass position i is

$$(S.F.)_{max} = \sum_{j=1}^{n} K_{ij} \, x_{j,\, max} \qquad (6.31)$$

6.4 LOAD DISTRIBUTION BETWEEN SHEAR WALL AND FRAME :

The external shear in any horizontal plane of a structure is distributed to the various resisting elements in proportion to their stiffness[17,18,19]. Therefore the shear V_i at an element i, at any particular level, is obtained as :

$$V_i = (K_i / \sum K_i)\, V \quad (\text{ i = for all members}) \tag{6.32}$$

$$K_i = 12\, E\, I_i\, \varepsilon_i / L_i^3 \tag{6.33}$$

in which V is the total shear at that level, $E\,I_i$ is the flexural rigidity, L_i is the span length, and ε_i is the shear parameter given by[20].

$$\varepsilon_i = 1 / (1+12\, k\, E\, I_i / L_i^2\, A\, G) \tag{6.34}$$

The distribution of the external shears at each storey in proportion to the stiffnesses, involves an approximation as the interaction with other storeys is disregarded.

CHAPTER VII

ILLUSTRATED NUMERICAL EXAMPLES

NUMERICAL EXAMPLE 1

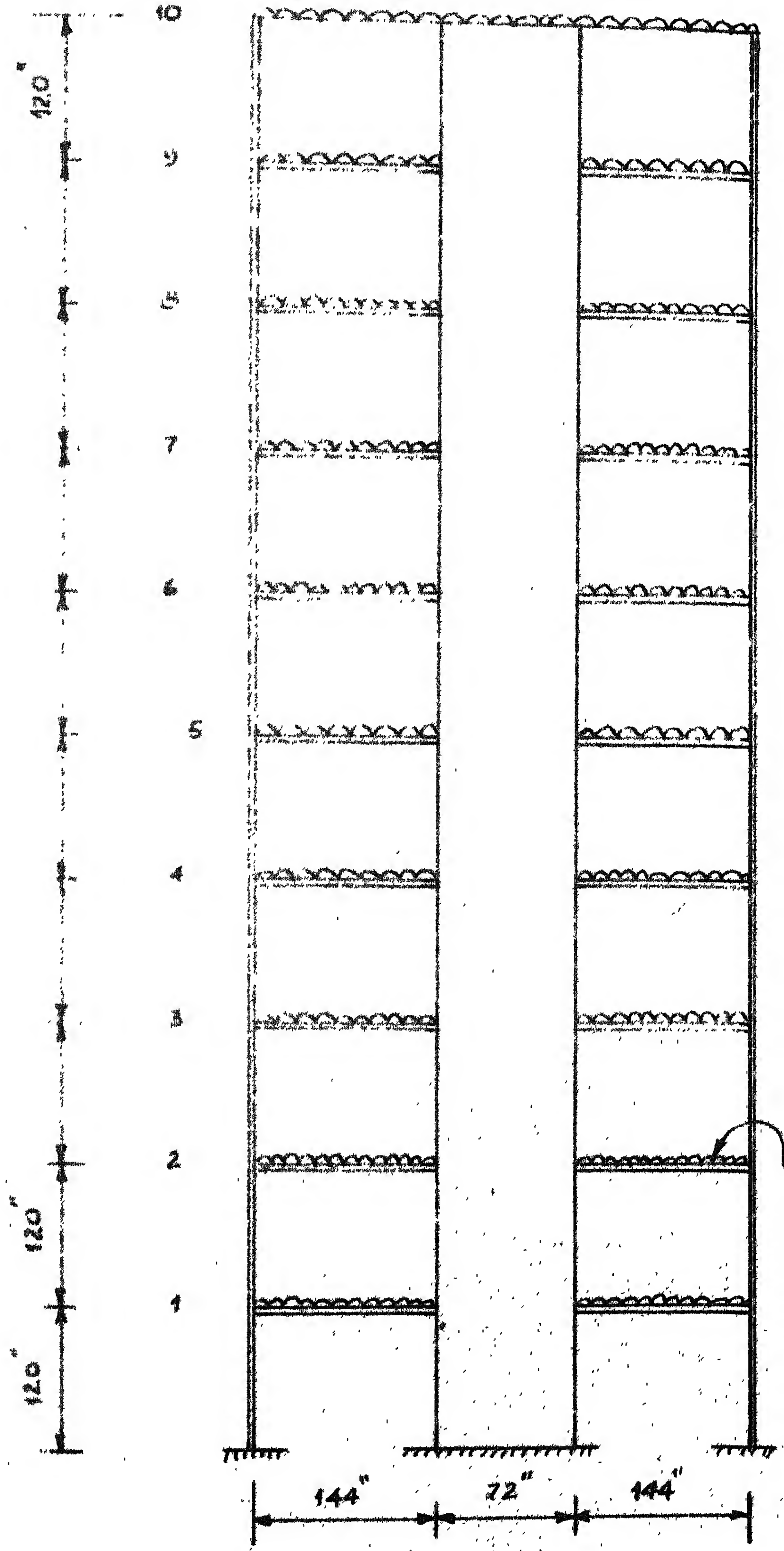
A 10 - STOREY SHEARWALL STRUCTURE TYPE I

7.1 METHOD I :

Consider a shearwall structure shown in Fig. 7.1. The weights on the floors and of walls etc. are shown on the building. The value of E is taken as 3000 kip/in^2. The building consists of series of such frames spaced at 20 feet interval. It is assumed that both structural properties and loading are uniform along the length of the building.

7.1.1 FORMATION OF MASS MATRIX :

Density of structural material = 150 lbs/cft. The weights of each storey including that of floor, columns and walls is calculated. The weights of 1st to 9th storey come out to be the same but the top storey has got less



DIMENSIONS :-

COLUMNS = 18" x 12"
BEAMS = 12" x 18"
WALL = 72" x 12"

100psf (INCLUDING BEAM)
LENGTH OF BAY = 20'
I FOR WALL = 373248 IN
I FOR COLUMNS = 5832
I FOR BEAMS = 5832 IN
E = 3000 KIP
20 psf FOR WALLS

G. 7.1.1
S & DETAIL OF LOADING

L STRUCTURE TYPE I

SHEARWALL STRUCTURE TYPE I & II
MASS MATRIX (DIAGONAL)

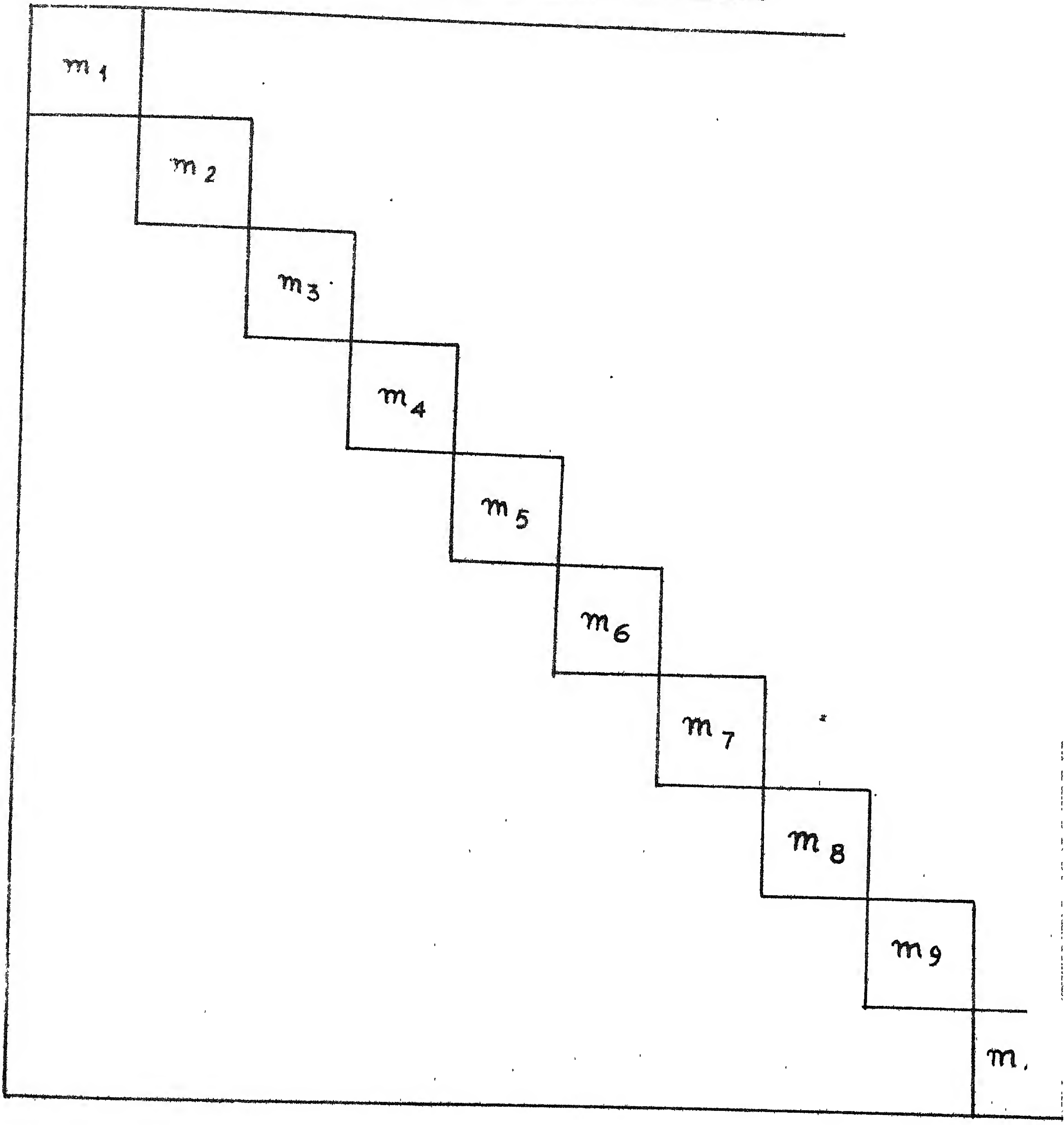


FIG. 7·1.2.

TYPE I $\quad m_1$ TO $m_9 = 0{\cdot}224$ KIP-SEC.^{2}/IN.

$m_{10} = 0{\cdot}1985$

TYPE II $\quad m_1$ TO $m_9 = 0{\cdot}485$

$m_{10} = 0{\cdot}1545$

SHEARWALL STRUCTURE TYPE I & II

MASS MATRIX (DIAGONAL)

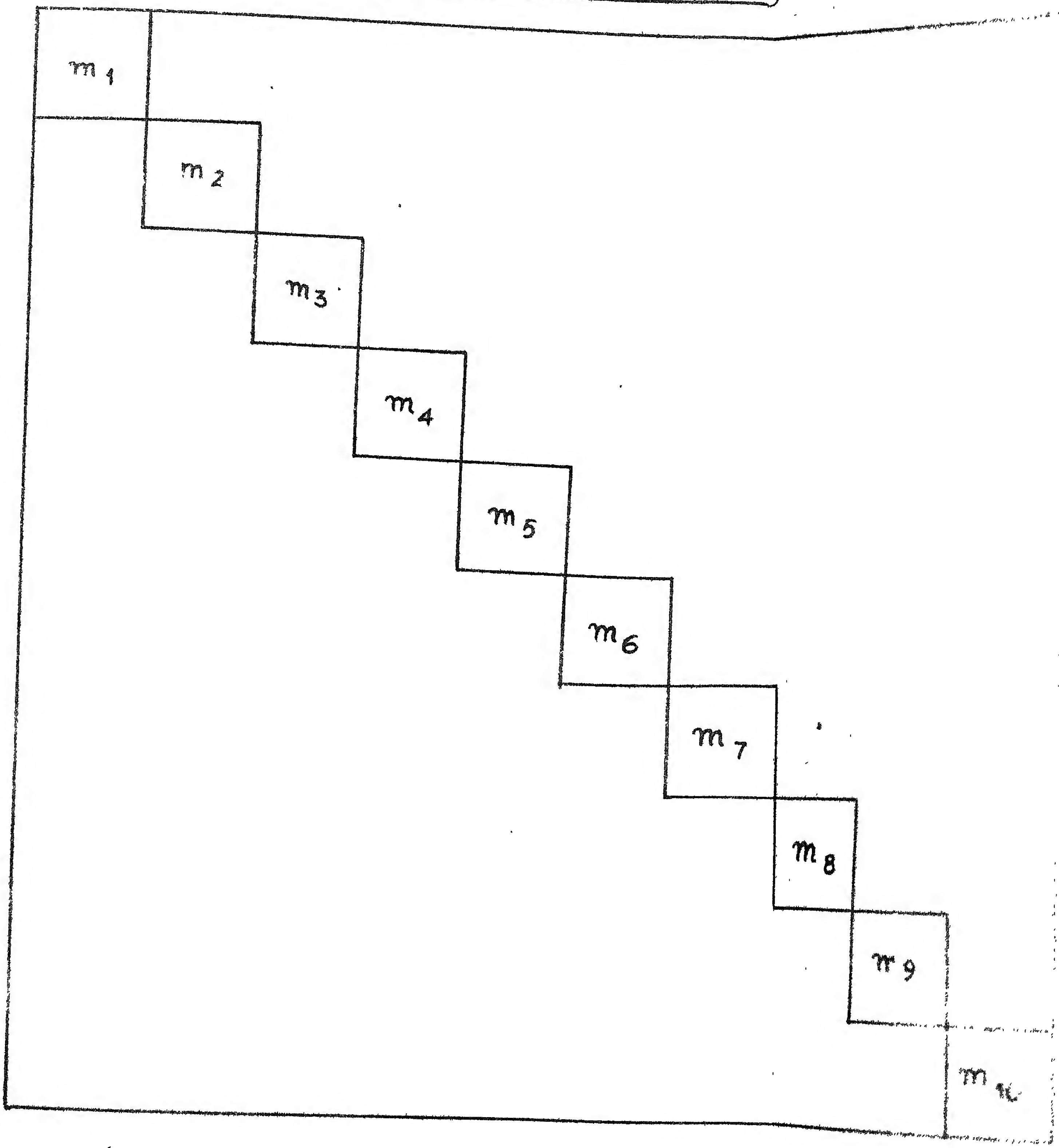


FIG. 7·1.2.

TYPE I m_1 TO m_9 = 0·224 KIP-SEC.^{2}/IN.

m_{10} = 0·1985

TYPE II m_1 TO m_9 = 0·185

m_{10} = 0·1545

weight. By dividing the weight of each storey by gravitational acceleration (g) we get the value of mass of each storey.

The diagonal mass matrix is shown in Fig. 7.1.2.

7.1.2 STIFFNESS MATRIX A :

The square matrix A is of the form shown in Fig. 2.5 and is of the order 40. It consists of submatrices B, B', C, D, E, F, G, H, I, J, K & L. The elements in these submatrices are as follows :-

$$[B] = \begin{bmatrix} 0.1652400E+07 & 0.42525000E+06 \\ 0.42525000E+06 & 0.76532849E+08 \end{bmatrix}$$

$$[B'] = \begin{bmatrix} 0.10692000E+07 & 0.42525000E+06 \\ 0.42525000E+06 & 0.39208050E+08 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 0.16524000E+07 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 0.10692000E+07 \end{bmatrix}$$

$$[E] = \begin{bmatrix} 0.42525000E+06 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 0.29160000E+06 \end{bmatrix}$$

$$[G] = \begin{bmatrix} 0.188662400E+08 \end{bmatrix}$$

$$[H] = \begin{bmatrix} 0.72899999E+04 & 0.46656000E+06 & 0.72899999E+04 \end{bmatrix}$$

$$[I] = \begin{bmatrix} -0.72899999E+04 & 0.46656000E+06 & 0.72899999E+04 \end{bmatrix}$$

$$[J] = \begin{bmatrix} 0.16038000E+05 & -0.80189999\ E+04 \\ -0.80189999E+04 & 0.16038000E+05 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 0.16038000E+05 & -0.80189999E+04 \\ -0.80189999E+04 & 0.80189999E+04 \end{bmatrix}$$

$$\& \ [L] = \begin{bmatrix} -0.80189999E+04 \end{bmatrix}$$

7.1.3 LATERAL STIFFNESS MATRIX $[K]$:

The matrix $[K]$ is a symmetric square matrix of the order 10. The arrangements of elements is shown in Fig. 4.1. The elements of left half of the matrix are given in Table 7.1.1.

7.1.4 FREQUENCIES, TIME PERIODS AND MODE SHAPES :

The frequencies, time periods and mode shapes are given in the table 7.1.2. The mode shapes are drawn in Fig. 7.1.2.

7.1.5 WIND ANALYSIS :

Wind loading has been taken as shown in Fig. 6.2.

t_r = 0.1 Sec and F = 1.0 Kip.

The values of $(DLF)_{max}$ are found from the graph of $\frac{t_r}{T}$ Vs $(DLF)_{max}$ as given in the Table 7.1.3. The modal static deflections are listed in the Table 7.1.4. The table 7.1.5 gives the values of upper and lower bounds of deflection and shear force at each storey level.

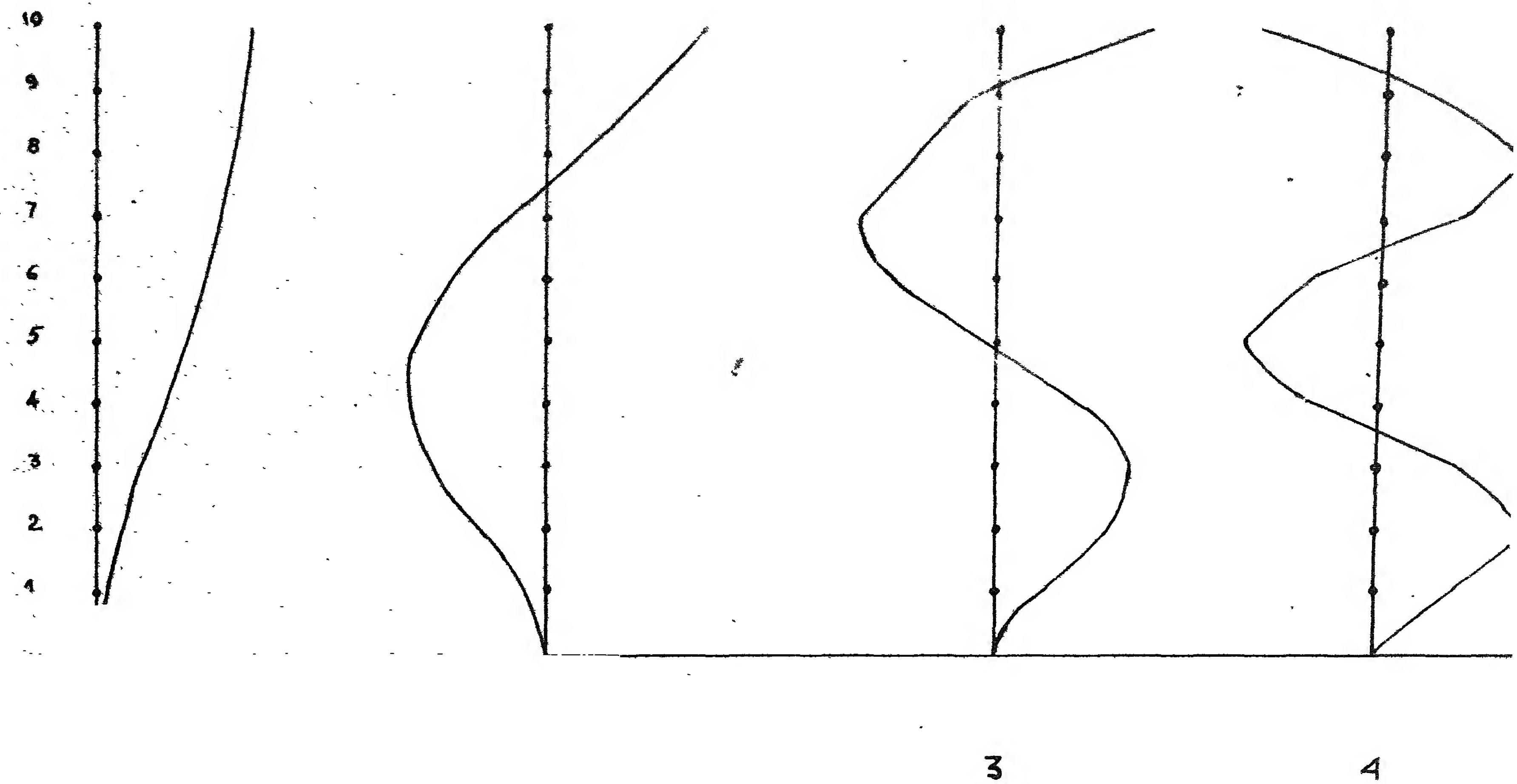
7.1.6 EARTHQUAKE ANALYSIS (DAMPING RATIO = 0.00) :

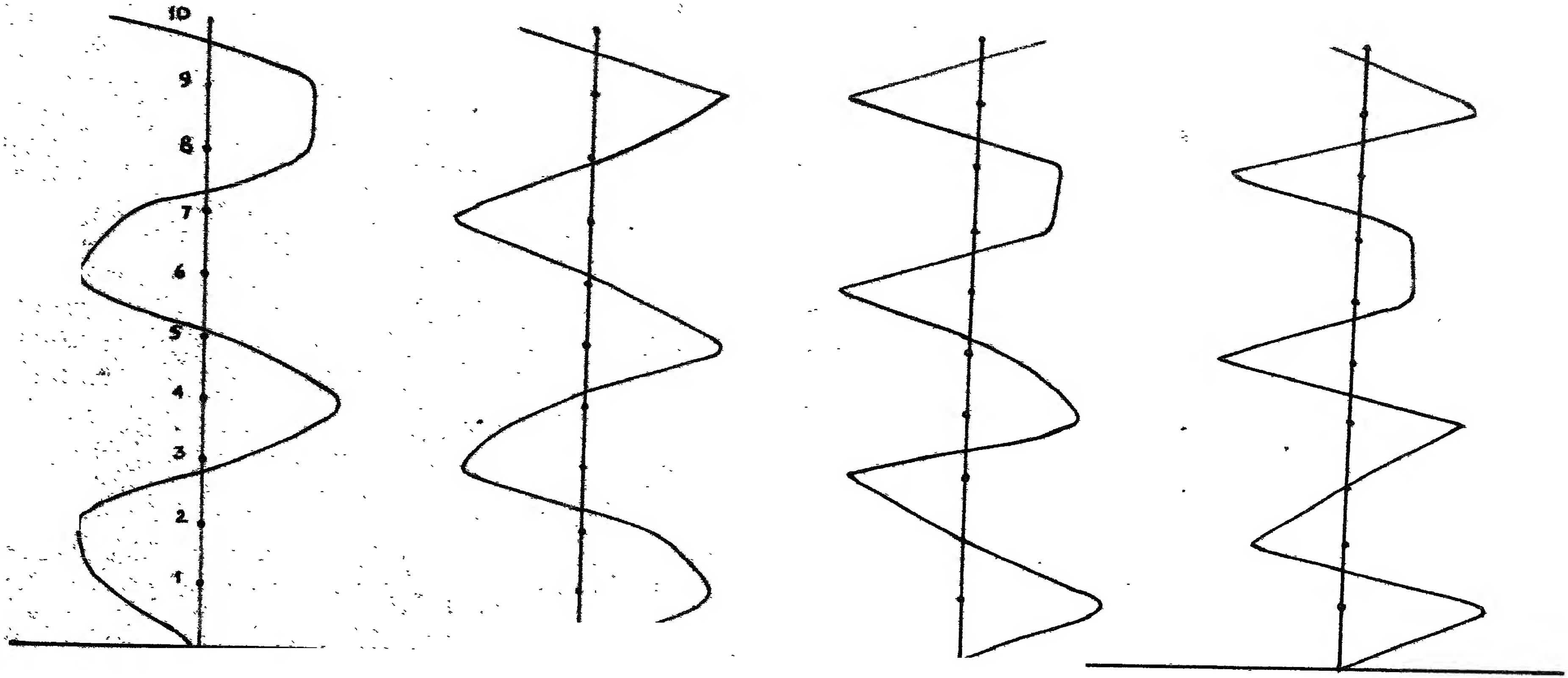
The structure has been subjected to the Koyna Earthquake of December 11,1967. The magnitude of this shock was around 6.5 with its epicentre at a distance of about 3 miles from the Koyna Dam. The depth of focus was about 15 miles The shock recorded a peak ground acceleration of 0.63 g having duration of strong ground motion of about 10 seconds.

SHEARWALL STR TURE TYPE I

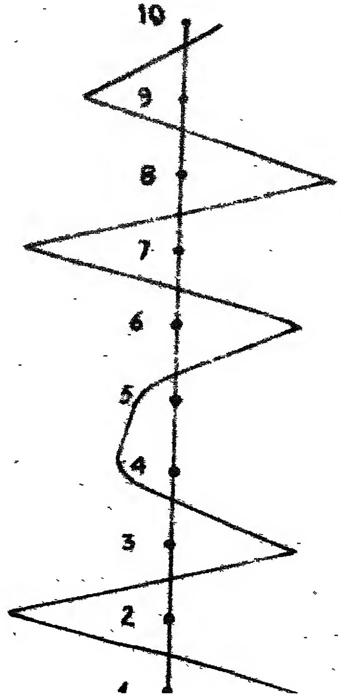
MODE SHAPES

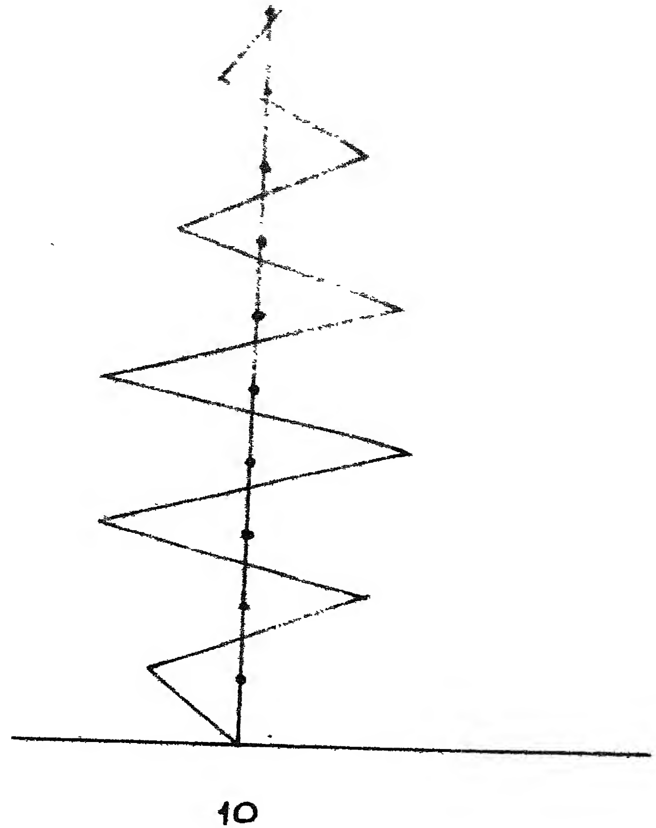
FIG.7.i.3





8


10
9
8
7
6
5
4
3
2
1


10

From the velocity spectrum of Koyna Earthquake, we get the values S_V for first five modes. The modal participation factors, values of S_V and the upper and lower bounds of deflection & shear force at each storey level have been given in the tables 7.15 and 7.16.

7.17 EARTHQUAKE ANALYSIS (DAMPING RATIO = 0.02) :

The values of S_V, MPF_S, the upper and lower bounds of deflection and shear force at each storey level are listed in Tables 7.17 and 7.18.

7.18 SHEAR DISTRIBUTION BETWEEN SHEARWALL AND FRAME :

Applying equations (6.32,6.33 & 6.34) it was found that the shear resisted by shearwall at each storey level is 94.8 percent of the total shear and the rest 5.2 percent is resisted by both the columns. The tables 7.15(a), 7.17 (a) and 7.1.9 (a) give the values of shear distribution between the wall and columns due to wind and earthquake loading.

7.1.9 METHOD II :

The same problem has been solved by the method illustrated in Chapter III.

7.1.10 FORMATION OF MASS MATRIX :

The mass matrix is of the same form as shown in Fig. 7.1.2 and the masses of storeys are the same.

7.1.11 STIFFNESS MATRIX [A] :

The matrix [A] is square and symmetric and of the order 60. Fig. 3.3 shows the arrangement of submatrices B,B', C, D, E,F, G, H and I and the arrangement of elements in these submatrices have been given in para 3.2. The coefficients a_i, b_i, c_i, d_i and e_i are given in the Table 7.1 10 as calculated for the shearwall structure Type I shown in Fig. 7.1.1.

7.1.12 LATERAL STIFFNESS MATRIX K :

The matrix [K] is a symmetric square matrix of order 10. The arrangement of elements is shown in Fig. 4.1.

The coefficients of matrix [K] are listed in Table 7.1.11.

7.1.13 FREQUENCIES, TIME PERIODS AND MODE SHAPES :

The frequencies, time periods and mode shapes are given in the Table 7.1.12. The plots of the mode shapes are same as shown in Fig. 7.1.2.

TABLE 7.1.1 COEFFICIENTS OF LATERAL STIFFNESS MATRIX K

i	j	K_{ij}	i	j	K_{ij}
1	1	12772.13696	5	5	9914.08520
2	1	-7965.91800	6	1	- 52.83041
2	2	10104.11848	6	2	190.91865
3	1	3048.08964	6	3	-734.43724
3	2	-7281.14728	6	4	2858.05612
3	3	9926.99816	6	5	-7231.48968
4	1	- 783.66887	6	6	9913.23320
4	2	2870.96936	7	1	13.79866
4	3	-7235.07952	7	2	-49.66670
4	4	9914.96664	7	3	190.03753
5	1	202.95076	7	4	-734.01111
5	2	- 737.60097	7	5	2857.20392
5	3	2858.93748	7	6	-7228.44896
5	4	-7231.91576	7	7	9901.70528

TABLE 7.1.1 (Continued)

i	j	K_{ij}	i	j	K_{ij}
8	1	- 3.59878	9	7	2667.66112
8	2	12.91753	9	8	-6540.29064
8	3	-49.24057	9	9	7252.86360
8	4	189.18525	10	1	- 0.14728
8	5	- 730.97042	10	2	0.52739
8	6	2845.67628	10	3	- 2.00373
8	7	-7184.49328	10	4	7.66239
8	8	9733.69016	10	5	- 29.40904
9	1	0.88542	10	6	113.42132
9	2	- 3.17264	10	7	-440.40540
9	3	12.06526	10	8	1726.24114
9	4	- 46.19991	10	9	-2844.23748
9	5	177.65763	10	10	1468.31450
9	6	- 687.01470			

TABLE 7.1.2 FREQUENCIES, TIME PERIODS AND MODE SHAPES

S.L.	1	2	3	4	5
FREQUENCY Radian/Sec.	6.18732	20.88347	42.12405	72.13858	111.51777
TIME PERIOD Sec.	1.01590	0.30099	0.14922	0.08713	0.05637
MODE SHAPE	0.02189	-0.07365	0.14635	0.23466	- 0.32495
	0.07206	-0.21996	0.36807	0.45069	-0.40721
	0.13514	-0.35600	0.43734	0.26183	0.10168
	0.20208	-0.42808	0.27761	-0.18129	0.43372
	0.26721	-0.40906	-0.03275	-0.43009	0.06723
	0.32681	-0.29772	-0.31961	-0.22957	-0.40808
	0.37847	-0.11527	-0.41724	0.20774	-0.23084
	0.42091	0.10247	-0.26202	0.41750	0.30933
	0.45411	0.31718	0.07881	0.15442	0.32221
	0.48001	0.50581	0.471180	-0.40175	-0.32603

Sl.No.	6	7	8	9	10
FREQUENCY Radian/Sec.	159.80176	215.21389	273.57136	327.02454	364.86414
TIME PERIOD Sec.	0.03933	0.02921	0.02298	0.01922	0.01723
	0.40288	0.45428	− 0.46252	0.40334	− 0.24706
	0.22818	− 0.03790	0.29669	− 0.42809	0.32488
MODE SHAPE	−0.39695	− 0.38316	0.04697	0.33910	−0.38818
	−0.13833	0.34508	−0.36084	−0.14192	0.42052
	0.42662	0.11213	0.42102	−0.09966	−0.42028
	−0.05158	−0.43325	−0.18558	0.30999	0.38741
	−0.43774	0.22928	−0.18009	−0.42301	−0.32446
	0.03358	0.25476	0.42028	0.40375	0.23650
	0.40820	−0.41511	−0.35658	−0.25311	−0.12820
	−0.25472	0.19032	0.13318	0.08295	0.03912

TABLE 7.1.3 DYNAMIC LOAD FACTORS $(DLF)_{max.}$

WIND ANALYSIS (t_r = 0.1 Sec. F = 1.0 Kip.

MODE	t_r Sec	T Sec.	t_r/T	$(DLF)_{max.}$
1	0.1	1.01590	0.985	1.08
2	0.1	0.30099	0.333	1.810
3	0.1	0.14922	0.666	1.400
4	0.1	0.08713	1.147	1.150
5	0.1	0.05637	1.770	1.120

TABLE 7.1.4 MODAL STATIC DEFLECTIONS

MODE	1	2	3	4	5
MODAL STATIC DEFLECT-ION	0.33036193	-0.01027232	0.00193174	0.00042310	-0.00016822

TABLE 7.1.5 UPPER AND LOWER BOUNDS OF DEFLECTIONS AND SHEAR FORCES(S.F.)

STOREY			1	2	3	4	5
DEFLECTION (in)	BOUND	UPPER	0.00975	0.03109	0.05617	0.08098	0.10325
		LOWER	0.00794	0.02605	0.04868	0.07254	0.09564
S.F. (Kips)	BOUND	UPPER	19.4295	18.7936	17.2882	15.5633	13.5406
		LOWER	11.0925	10.7908	10.0940	9.1838	8.1179

Contd. Table 7.1.5

STOR			6	7	8	9	10
	BOUND	UPPER	0.12319	0.13845	0.15305	0.16827	0.18220
		LOWER	0.11674	0.13506	0.15019	0.16213	0.17152
	BOUND	UPPER	11.6652	9.4344	7.3788	5.2288	2.9168
		LOWER	6.9851	5.7696	4.4985	3.1413	1.6179

TABLE 7.1.6 EARTHQUAKE ANALYSIS (DAMPING RATIO = 0.00)

MODAL PARTICIPATION FACTORS (MPF) AND S_v

MODE	1	2	3	4	5
TIME PERIOD (Sec.)	1.01590	0.30099	0.14922	0.08713	0.05637
MPF	0.44880	-0.05089	0.01692	0.00748	-0.00387
S_v (in/Sec)	16.55	13.00	24.40	15.80	7.50

TABLE 7.1.7 UPPER AND LOWER BOUNDS OF DEFLECTIONS AND SHEAR FORCES (S.F.)

STOREY			1	2	3	4	5
DEFLE-CTION	BOUND	UPPER	0.3089	0.8978	1.4538	1.9328	2.3217
		LOWER	0.1825	0.5777	1.0472	1.5320	2.0038
SHEAR FORCE	BOUND	UPPER	1440.034	1351.852	1177.188	1029.708	883.620
		LOWER	747.029	698.728	604.820	520.469	450.762

(Continued Table 7.1.7)

STOREY			6	7	8	9	10
DEFLE-CTION	BOUND	UPPER	2.7953	3.0909	3.3607	3.6429	4.1517
		LOWER	2.4392	2.8176	3.1294	3.3797	3.5866
SHEAR FORCE	BOUND	UPPER	770.084	612.946	465.626	306.776	197.026
		LOWER	383.234	308.118	227.536	146.665	96.070

TABLE 7.1.8 EARTHQUAKE ANALYSIS (DAMPING RATIO = 0.02)

MODAL PARTICIPATION FACTORS (MPF) AND S_v

MODE	1	2	3	4	5
TIME PERIOD	1.01590	0.30099	0.14922	0.08713	0.05637
MPF	0.44879887	-0.05089285	0.01691932	0.00748274	-0.003865
S_v	13.70	7.33	9.06	7.10	3.54

TABLE 7.1.9 UPPER AND LOWER BOUNDS OF DEFLECTIONS AND SHEAR FORCE

STOREY			1	2	3	4	5
DEFLE-CTION	BOUND	UPPER	0.2014	0.6111	1.0461	1.4603	1.8243
		LOWER	0.1398	0.4548	0.8442	1.2535	1.6502
SHEAR FORCE	BOUND	UPPER	702.628	663.408	585.371	518.671	447.771
		LOWER	349.219	327.969	287.906	253.201	220.920

Continued Table 7.1.9

STOREY			6	7	8	9	10
DEFLE-CTION	BOUND	UPPER	2.1872	2.4482	2.6928	2.9351	3.2382
		LOWER	2.0131	2.3283	2.5887	2.7947	2.9584
SHEAR FORCE	BOUND	UPPER	387.575	310.213	238.955	159.498	97.265
		LOWER	187.143	151.942	115.808	76.247	44.996

TABLE 7.1.5 (a)

DISTRIBUTION OF SHEAR FORCE (S.F.) DUE TO WIND

STOREY	TOTAL SHEAR FORCE (KIPS) BOUND UPPER	TOTAL SHEAR FORCE (KIPS) BOUND LOWER	S.F. RESISTED BY SHEARWALL BOUND UPPER	S.F. RESISTED BY SHEARWALL BOUND LOWER	S.F. RESISTED BY COLUMNS BOUND UPPER	S.F. RESISTED BY COLUMNS BOUND LOWER
10	2.9168	1.6179	2.7651	1.5338	0.1517	0.0841
9	5.2288	3.1413	4.9569	2.9780	0.2719	0.1633
8	7.3788	4.4985	6.9951	4.2646	0.3837	0.2339
7	9.4344	5.7696	8.9438	5.4696	0.4906	0.3000
6	11.6652	6.9851	11.0586	6.6219	0.6066	0.3632
5	13.5406	8.1179	12.8365	7.6958	0.7041	0.4221
4	15.5633	9.1838	14.7540	8.7062	0.8093	0.4776
3	17.2882	10.0940	16.3892	9.5691	0.8990	0.5249
2	18.7936	10.7908	17.8163	10.2297	0.9773	0.5611
1	19.4295	11.0925	18.4192	10.5157	1.0103	0.5768
BASE	19.4295	11.0925	18.4192	10.5157	1.0103	0.5768

TABLE 7.1.7 (a)

DISTRIBUTION OF SHEAR FORCE (S.F.) DUE TO EARTHQUAKE (DAMPING RATIO = 0.00)

STOREY	TOTAL S.F. KIPS		S.F. RESISTED BY SHEARWALL		S.F. RESISTED BY COLUMNS	
	BOUND		BOUND		BOUND	
	UPPER	LOWER	UPPER	LOWER	UPPER	LOWER
10	197.026	96.070	186.781	91.074	10.245	4.996
9	306.776	146.665	290.824	139.038	15.952	7.627
8	465.626	227.536	441.413	215.704	24.213	11.832
7	612.946	308.118	581.073	292.096	31.873	16.022
6	770.084	383.234	730.040	363.306	40.044	19.920
5	883.620	450.762	837.672	427.322	45.948	23.440
4	1029.708	520.469	976.163	493.405	53.545	27.064
3	1177.188	604.820	1115.974	573.369	61.214	31.451
2	1351.852	698.728	1281.556	662.394	70.296	36.334
1	1440.034	747.029	1365.152	708.183	74.882	38.846
BASE	1440.034	747.029	1365.152	708.183	74.882	38.846

TABLE 7.1.9 (a)

DISTRIBUTION OF SHEARFORCE (S.F.) DUE TO EARTHQUAKE (DAMPING RATIO = 0.02)

STOREY	TOTAL S.F. KIPS		S.F. RESISTED BY SHEARWALL		S.F. RESISTED BY COLUMNS	
	BOUND		BOUND		BOUND	
	UPPER	LOWER	UPPER	LOWER	UPPER	LOWER
10	97.265	44.996	92.207	42.656	5.058	2.340
9	159.498	76.247	151.204	72.282	8.294	3.965
8	238.955	115.808	226.529	109.786	12.426	6.022
7	310.213	151.942	294.082	144.041	16.131	7.901
6	387.575	187.143	367.421	177.412	20.154	9.731
5	447.771	220.920	424.487	209.432	23.284	11.488
4	518.671	253.201	491.700	240.035	26.971	13.166
3	585.371	287.906	554.932	272.935	30.439	14.971
2	663.408	327.969	628.911	310.915	34.497	17.054
1	702.628	349.219	666.091	331.060	36.537	18.159
BASE	702.628	349.219	666.091	331.060	36.537	18.159

TABLE 7.1.10 STIFFNESSES FOR SHEAR WALL STRUCTURE 1

i	d_i	b_i	c_i	d_i	e_i
1	0.10692000E+07	0.56024999E+04	0.42525000 E+06	0.72899999 E+04	0.29160000 E+06
2	0.54703125E+04	0.75937499E+04	0.75937499 E+04	0.46656000 E+06	0.54000000 E+04
3	0.39208050E+07	0.42525000E+06		0.72899999 E+04	0.18662400 E+08
4	0.10692000E+07	0.50624999E+04		0.80189999 E+04	0.29160000 E+06
5	0.54703125E+04				0.54000000 E+04
6	0.80189999E+04				
7	0.16524000E+07				
8	0.10870312E+05				
9	0.76532849E+08				
10	0.16524000E+07				
11	0.10870312E+05				
12	0.16038000E+05				

K_1	0.17496000E+08
K_2	0.17496000E+08
K_3	0.17496000E+08
K_4	0.17496000E+08
K_5	0.64800000E+06
K_6	0.64800000E+06
K_7	0.11197440

TABLE 7.1.11 COEFFICIENTS OF LATERAL STIFFNESS MATRIX K

i	j	K_{ij}	i	j	K_{ij}
1	1	12770.53699	5	5	9912.07336
2	1	-7966.43378	6	1	- 52.74690
2	2	10102.20386	6	2	191.04607
3	1	3048.59500	6	3	-734.44997
3	2	-7281.58502	6	4	2858.48038
3	3	9925.03210	6	5	-7231.96783
4	1	-783 .66055	6	6	9911.23022
4	2	2871.42392	7	1	13.82261
4	3	-7235.52869	7	2	-49.65409
4	4	9912.98010	7	3	190.09200
5	1	203.08791	7	4	-734.09695
5	2	-737.60748	7	5	2857.54712
5	3	2859.36685	7	6	-7228.98877
5	4	-7232.38574	7	7	9899.53882

ontd. (Table 7.1.11)

i	j	K_{ij}	i	j	K_{ij}
8	1	-3.36605	9	7	2676.53671
8	2	13.15719	9	8	-6540.43994
8	3	-49.00608	9	9	7242.21082
8	4	189.46995	10	1	2.31435
8	5	-730.81548	10	2	3.02807
8	6	2846.27530	10	3	0.56586
8	7	-7184.76923	10	4	10.31660
8	8	9731.84827	10	5	- 26.59214
9	1	0.02911	10	6	116 .21940
9	2	-4.04320	10	7	-436.65690
9	3	11.17310	10	8	1726.64655
9	4	-47.13009	10	9	-2826.33197
9	5	176.70089	10	10	1429.22980
9	6	-688.06203			

TABLE 7.1.12 MODES, TIME PERIODS AND MODE SHAPES

S.No.	1	2	3	4	5
FREQUENCY Radius/Sec.	5.49297	19.62517	41.63351	71.76955	111.32947
TIME PERIOD Sec.	1.14432	0.32029	0.15098	0.08758	0.05646
	-0.01875	-0.07212	0.14634	0.23464	-0.32519
	-0.06294	-0.21621	0.036989	0.45143	0.40855
MODE SHAPES	-0.12047	-0.35245	0.44363	0.26392	0.09996
	-0.18401	-0.42904	0.28898	-0.17969	0.43388
	-0.24875	-0.41875	-0.01942	-0.43232	0.06912
	-0.31132	-0.31800	-0.31025	-0.23612	-0.40807
	-0.36934	-0.14391	-0.41710	0.20165	- 0.23415
	-0.42131	0.07210	-0.27175	0.41804	0.30686
	-0.46673	0.29476	0.06463	0.16202	0.32448
	-0.50672	0.50048	0.46132	-0.39556	-0.32174

Continued Table 7.1.12

S.No.					
FREQUENCY Radius/Sec.	159.66274	215.13933	273.52383	327.00605	364.85893
TIME PERIOD Sec.	1.14432 0.03937	0.32029 0.02922	0.15098 0.02298	0.01922	0.01723
	0.40295	0.45445	-0.46258	0.40344	-0.24710
	0.22855	-0.03750	0.29667	-0.42810	0.32494
MODE SHAPE	-0.39710	-0.38340	0.04725	0.33907	-0.38822
	-0.13949	0.34494	-0.36103	-0.14175	0.42057
	0.42647	0.11291	0.42105	-0.09984	-0.42029
	0.05287	-0.43339	0.18515	0.31021	0.38741
	-0.43822	0.22669	-0.18055	-0.42308	-0.32439
	0.03143	0.25577	0.42059	0.40371	0.23641
	0.40874	-0.41489	-0.35620	-0.25283	-0.12804
	-0.25196	0.18887	0.13226	0.08251	0.03891

TABLE 7.1.13 WIND ANALYSIS (t_r=0.1 Sec. F = 1.0 Kip)

DYNAMIC LOAD FACTORS $(DLF)_{max.}$

MODE	t_r Sec.	T Sec	t_r/T	$(DLF)_{max.}$
1	0.1	1.14432	0.858	1.18
2	0.1	0.32029	0.312	1.86
3	0.1	0.15098	0.663	1.400
4	0.1	0.08758	0.141	1.125
5	0.1	0.05646	1.770	1.12

TABLE 7.1.14 MODAL STATIC DEFLECTIONS

MODE	1	2	3	4	5
MODAL STATIC DEFLEC-TION	-0.41309	-0.01292	0.00200	0.00043	-0.00017

TABLE 7.1.15 UPPER AND LOWER BOUNDS OF DEFLECTIONS AND SHEAR FORCES

STOREY			1	2	3	4	5
DEFLE-CTION	BOUND	UPPER	0.01146	0.03721	0.06858	0.10099	0.13160
		LOWER	0.00931	0.03114	0.05934	0.09029	0.12129
SHEAR FORCE	BOUND	UPPER	20.5877	19.9423	18.3956	16.5941	14.4620
		LOWER	11.9405	11.6323	10.9076	9.9374	8.7915

(Continued Table 7.1.15)

STOREY	BOUND		6	7	8	9	10
DEFLEC-TION	BOUND	UPPER	0.16046	0.18480	0.20812	0.23491	0.26057
		LOWER	0.15195	0.18007	0.20538	0.22762	0.24730
SHEAR FORCE	BOUND	UPPER	12.4697	10.0916	7.8987	5.6711	3.1865
		LOWER	7.5695	6.2728	4.9391	3.4845	1.8130

TABLE 7.1.16 MODAL PARTICIPATION FACTORS (MPF) AND S_v

MODE	1	2	3	4	5
TIME PERIOD	1.14432	0.32029	0.15098	0.08758	0.05646
MPF	-0.49746	-0.05980	0.01732	0.00759	-0.00388
S_v (in/sec.)	15.75	10.80	24.40	15.80	7.90

TABLE 7.1.17 UPPER AND LOWER BOUNDS OF DEFLECTIONS AND SHEAR FORCES

STOREY			1	2	3	4	5
DEFLE-CTION	BOUND	UPPER	0.2934	0.8557	1.3937	1.8758	2.2815
		LOWER	0.1687	0.5387	0.9894	1.4734	1.9683
SHEAR FORCE	BOUND	UPPER	1395.152	1306.478	1132.828	989.163	846.412
		LOWER	738.211	689.117	594.499	510.080	440.114

Continued Table 7.1.17

STOREY			6	7	8	9	10
DEFLE-CTION	BOUND	UPPER	2.8165	3.1944	3.5219	3.9039	4.5454
		LOWER	2.4515	2.9[illegible]07	3.3037	3.6619	3.9883
SHEAR FORCE	BOUND	UPPER	741.006	588.332	444.695	289.865	188.046
		LOWER	374.255	300.389	221.015	140.232	92.687

TABLE 7.1.18 MODAL PARTICIPATION FACTORS (MPF) AND S_v

MODE	1	2	3	4	5
TIME PERIOD	1.14432	0.32029	0.15098	0.08758	0.05646
MPF	-0.49745965	-0.5980013	0.017323228	0.00758555	-0.00387915
S_v (in/Sec.)	6.18	7.50	9.06	7.10	3.54

TABLE 7.1.19 UPPER AND LOWER BOUNDS OF DEFLECTIONS AND SHEAR FORCES

STOREY							
DEFLECTION	BOUND	UPPER	0.1301	0.3784	0.6136	0.8191	0.9798
		LOWER	0.0712	0.2255	0.4089	0.5994	0.7878
SHEAR FORCE	BOUND	UPPER	621.213	528.328	506.332	443.015	377.296
		LOWER	318.928	297.635	257.506	222.785	191.176

Continued Table 7.1.19

STOREY			6	7	8	9	10
DEFLE-CTION	BOUND	UPPER	1.1667	1.2795	1.3969	1.5904	1.8803
		LOWER	0.9690	1.1392	1.2965	1.4410	1.5757

SHEAR FORCE	BOUND	UPPER	325.240	257.238	197.212	131.439	83.938
		LOWER	159.262	127.485	96.318	62.109	39.874

TABLE 7.1.15 (a)

DISTRIBUTION OF SHEAR FORCE (S.F.) DUE TO WIND

STOREY	TOTAL S.F. KIPS		S.F. RESISTED BY SHEARWALL		S.F.RESISTED BY COLUMNS	
	BOUND		BOUND		BOUND	
	UPPER	LOWER	UPPER	LOWER	UPPER	LOWER
10	3.1865	1.8130	3.0208	1.7187	0.1657	0.0943
9	5.6711	3.4845	5.3762	3.3033	0.2949	0.1812
8	7.8987	4.9391	7.4880	4.6823	0.4107	0.2568
7	10.0916	6.2728	9.5668	5.9466	0.5248	0.3262
6	12.4697	7.5695	11.8213	7.1759	0.6484	0.3936
5	14.4620	8.7915	13.7100	8.3343	0.7520	0.4572
4	16.5941	9.9374	15.7312	9.4207	0.8629	0.5167
3	18.3956	10.9076	17.4390	10.3404	0.9566	0.5672
2	19.9423	11.6323	18.9053	11.0274	1.0370	0.6049
1	20.5877	11.9405	19.5171	11.3196	1.0706	0.6209
BASE	20.5877	11.9405	19.5171	11.3196	1.0706	0.6209

TABLE 7.1.17 (a)

DISTRIBUTION OF SHEAR FORCE (S.F.) DUE TO EARTHQUAKE (DAMPING RATIO = 0.00)

STOREY	TOTAL S.F. KIPS		S.F.RESISTED BY SHEARWALL		S.F. RESISTED BY COLUMNS	
	BOUND		BOUND		BOUND	
	UPPER	LOWER	UPPER	LOWER	UPPER	LOWER
10	188.046	92.687	178.268	87.867	9.778	4.820
9	289.865	140.232	274.792	132.940	15.073	7.292
8	444.695	221.015	421.571	209.522	23.124	11.493
7	588.332	300.389	557.739	284.769	30.593	15.620
6	741.006	374.255	702.474	354.794	38.532	19.461
5	846.412	440.114	802.399	417.228	44.013	22.886
4	989.163	510.080	937.727	483.556	51.436	26.524
3	1132.828	594.499	1073.921	563.585	58.907	30.914
2	1306.478	689.117	1238.541	653.283	67.937	35.834
1	1395.152	738.211	1322.604	699.824	72.548	38.387
BASE	1395.152	738.211	1322.604	699.824	72.548	38.387

TABLE 7.1.19(a) DISTRIBUTION OF SHEAR FORCE DUE TO EARTHQUAKE
DAMPING RATIO =(0.02)

STOREY	TOTAL S.F. Kips		S.F. RESISTED BY SHEAR WALL		S.F. RESISTED BY COLUMNS	
	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND
10	83.938	39.874	79.573	37.801	4.365	2.073
9	131.439	62.109	124.604	58.879	6.835	3.230
8	197.212	96.318	186.957	91.309	10.255	5.009
7	257.238	127.485	243.862	120.856	13.376	6.629
6	325.240	159.262	308.328	150.980	16.912	8.282
5	377.296	191.176	357.677	181.235	19.619	9.941
4	443.015	222.785	419.978	211.200	23.037	11.585
3	506.332	257.506	480.003	244.116	26.329	13.390
2	582.328	297.635	552.047	282.158	40.281	15.477
1	621.213	318.928	588.910	302.344	32.303	16.584
BASE	621.213	318.928	588.910	302.344	32.303	16.584

NUMERICAL EXAMPLE 2

A 10 - STOREY SHEARWALL STRUCTURE TYPE II

7.2 METHOD I :

Consider a shearwall structure Type II shown in Fig. 7.2.1. The weights on the floor and of walls etc. are shown on the building. The value of E is taken as 3000 Kip/in^2. The building consists of series of such frames spaced at 20 feet interval. It is assumed that both structural properties and loading are uniform along the length of the building.

7.2.1 FORMATION OF MASS MATRIX :

Density of structural material = 150 lbs/cft. The weight of each storey including that of floor, beam and walls etc. is calculated. The weights of 1st to 9th storey come out to be the same but the top storey weighs comparatively less. By dividing the weight of each storey by gravitational acceleration (g) we get the value of mass of each storey.

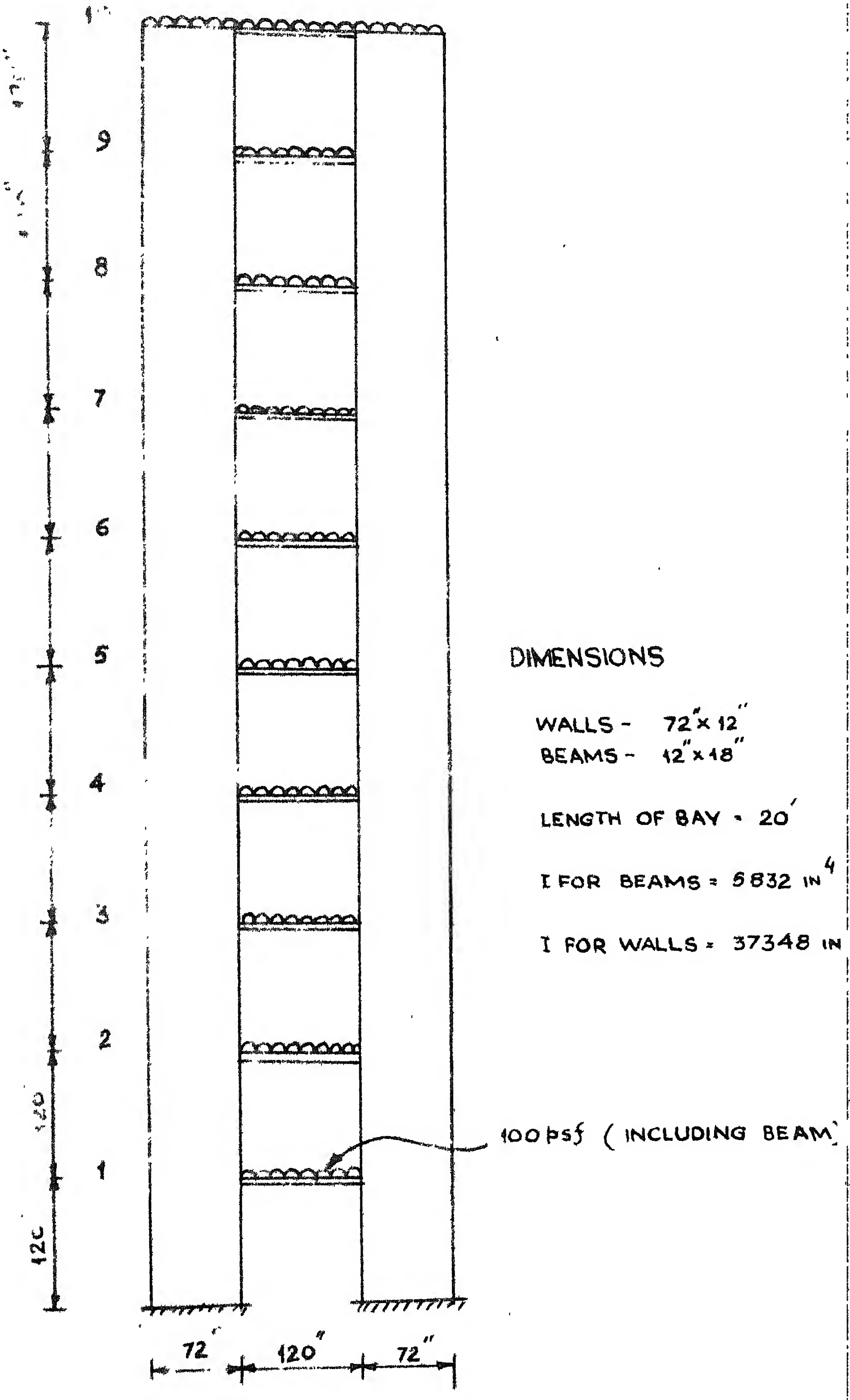


FIG 7.2.1.

DIMENSIONS & DETAIL OF LOADING.

SHEARWALL STRUCTURE TYPE II

The diagonal mass matrix is shown in Fig. 7.1.2.

7.2.2 STIFFNESS MATRIX [A] :

The square matrix A is of the form shown in Fig. 2.6 and is of the order 30. It consists of submatrices B, B', C, D, E, F, G and H. The elements of these submatrices are given below :

$$[B] = \begin{bmatrix} 0.75915142E{+}08 & 0.97394399E{+}06 \\ 0.97394399E{+}06 & 0.75915142E{+}08 \end{bmatrix}$$

$$[B'] = \begin{bmatrix} 0.38590343E{+}08 & 0.97394399E{+}06 \\ 0.97394399E{+}06 & 0.38590343E{+}08 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 0.18662400E{+}08 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 0.31104000E{+}05 & -0.155552000E{+}05 \\ -0.15552000E{+}05 & 0.31104000E{+}05 \end{bmatrix}$$

$$[E] = \begin{bmatrix} -0.15552000E{+}05 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 0.46656000E{+}06 & 0.46656000E{+}06 \end{bmatrix}$$

$$[G] = \begin{bmatrix} -0.46656000E{+}06 & -0.46656000E{+}06 \end{bmatrix}$$

$$[H] = \begin{bmatrix} 0.31104000E{+}05 & -0.15552000E{+}05 \\ -0.155552000E{+}05 & 0.15552000E{+}05 \end{bmatrix}$$

7.2.3 LATERAL STIFFNESS MATRIX $[K]$:

The matrix $[K]$ is a symmetric square matrix of order 10. The arrangement of elements is shown in Fig. 4.1. The elements of left half of the matrix are listed in Table 7.2.1.

7.2.4 FREQUENCIES, TIMEPERIODS AND MODE SHAPES :

The frequencies, time periods and mode shapes are given in the Table 7.2.2. The mode shapes are plotted in Fig. 7.2.2.

7.2.5 WIND ANALYSIS :

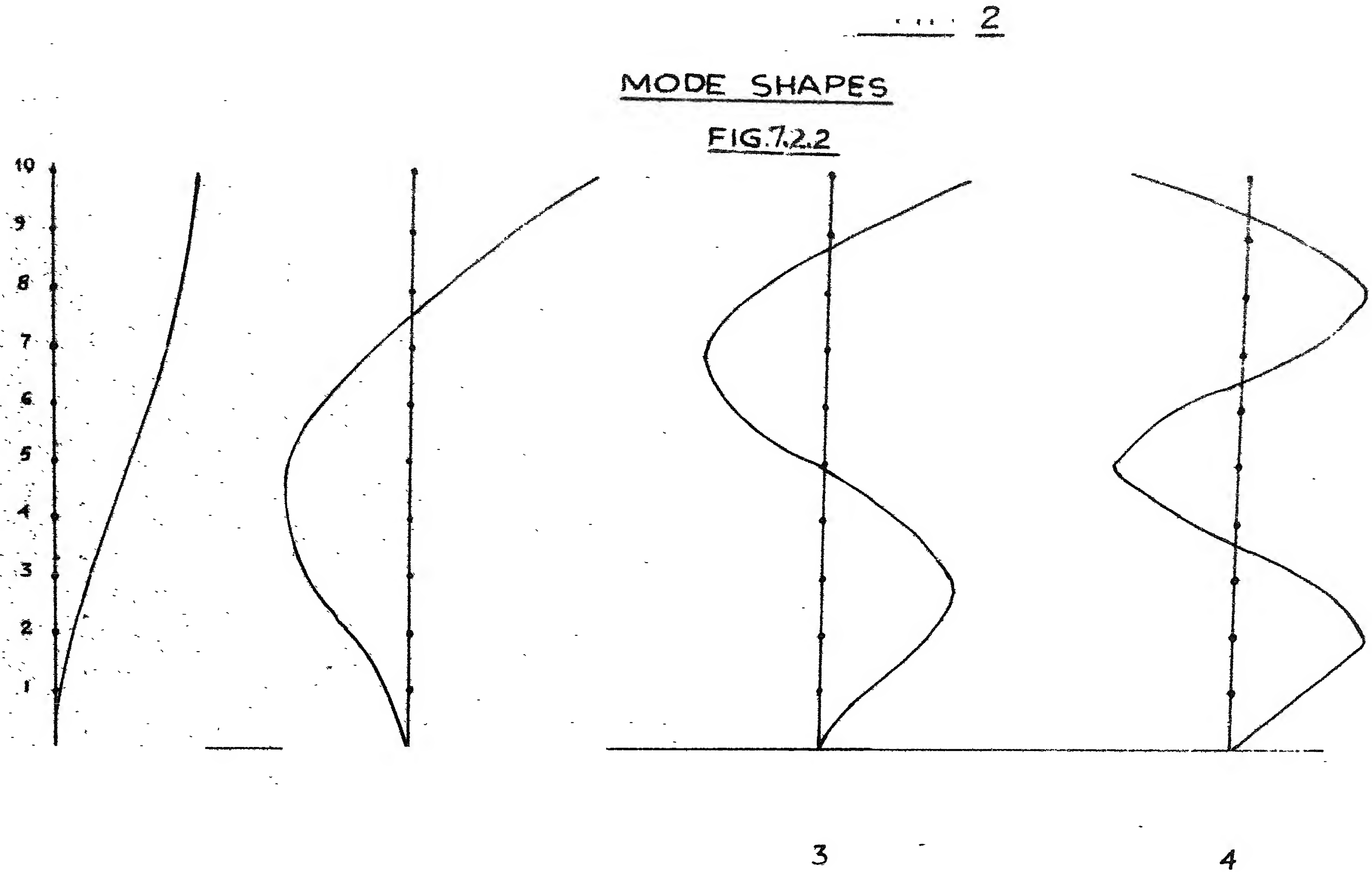
Wind loading has been taken as shown in Fig. 6.2.

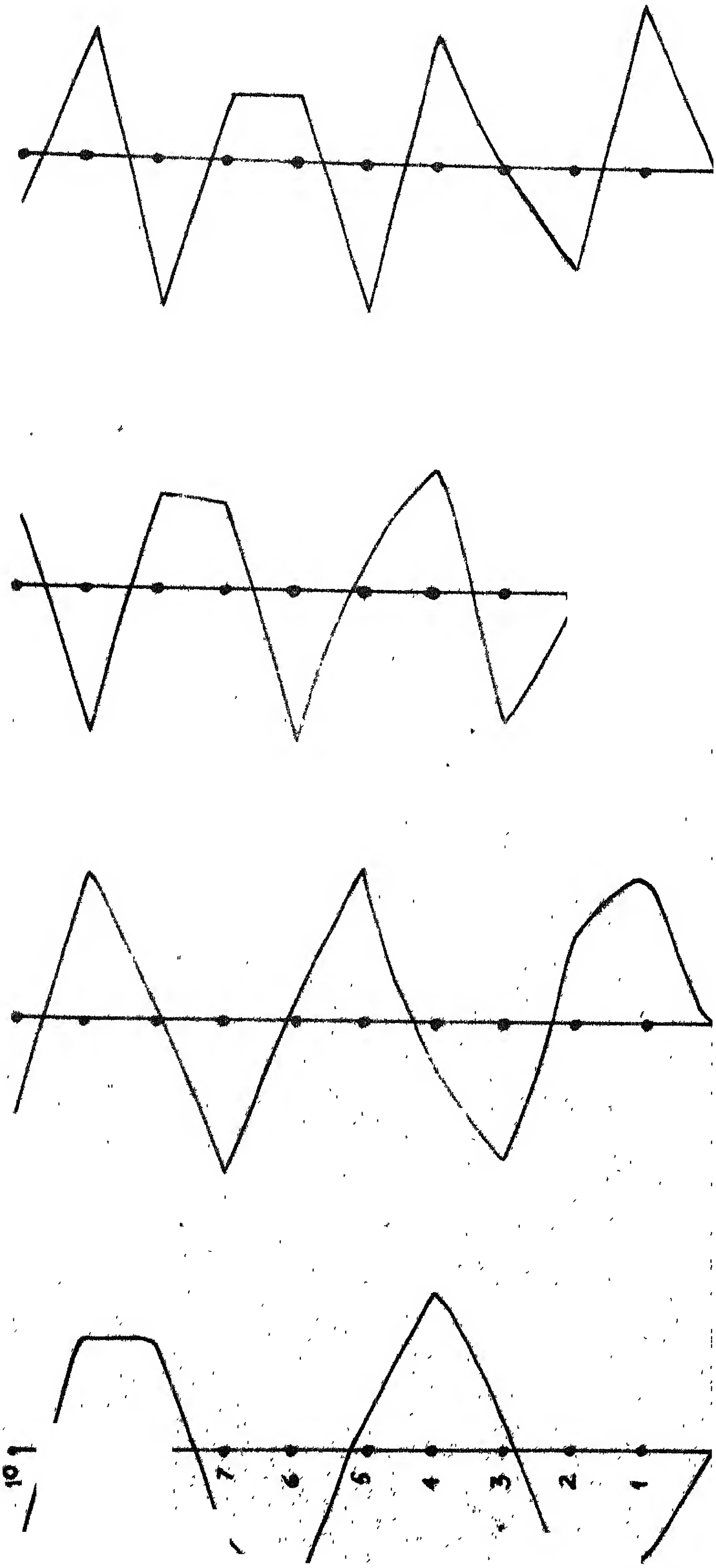
$t_r = 0.1$ sec and $F = 1.0$ Kip.

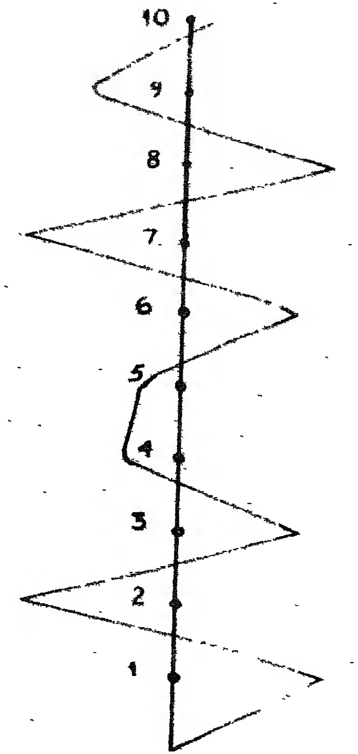
The values of $(DLF)_{max}$ are found from the graph of $\frac{t_r}{T}$ Vs. $(DLF)_{max}$ as given in the Table 7.2.3. The Table 7.2.4

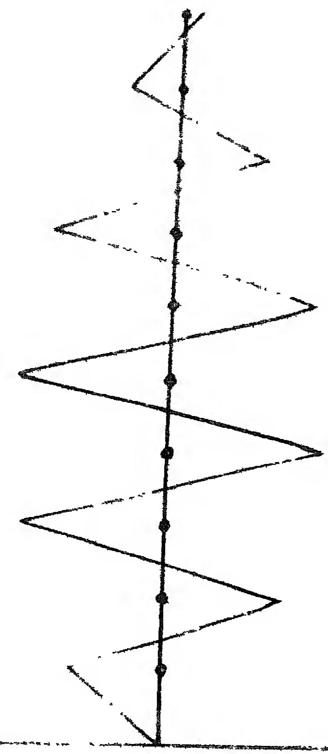
MODE SHAPES

FIG.7.2.2




10
7
6
5
4
3
2
1





10

and 7.2.5 gives modal static deflections and the values of upper and lower bounds of deflection and shear force at each storey level.

7.2.6 EARTHQUAKE ANALYSIS (DAMPING RATIO = 0.00) :

The structure has been subjected to the Koyna Earthquake of December 11,1967 (para 7.6).

From the velocity spectrum of the Koyna Earthquake, we get the values of S_v for first five modes. The modal participation factors, values of S_v and the upper and lower bounds of deflection and shear force have been given in the Tables 7.2.6 and 7.2.7.

7.2.7 EARTHQUAKE ANALYSIS (DAMPING RATIO) = 0.02):

The values of S_v, MPF, the upper and lower bounds of deflection and shear force at each storey level are given in Tables 7.2.8 and 7.2.9.

7.2.8 METHOD II :

The same numerical problem has been solved by the method illustrated in Chapter III.

7.2.9 FORMATION OF MASS MATRIX :

The mass matrix is of the same form as shown in Fig. 7.2 and the masses of storeys are the same as calculated in para 7.2.1.

7.2.10 STIFFNESS MATRIX A :

The matrix A is square and symmetric and of the order 50. Fig. 3.3, shows the arrangement of submatrices B, B', C, D, E, F, G, H and I and the arrangement of elements in these submatrices have been given in para 3.3. The coefficients a_i and b_i are given in the Table 7.2.10 as calculated for the shearwall structure Type II shown in Fig. 8.1.

7.2.11 LATERAL STIFFNESS MATRIX [K]:

The matrix [K] is a square symmetric matrix of order 10. The arrangement of elements in this matrix is shown in Fig. 4.1.

The left half of the coefficients of the matrix [K] are listed in Table 7.2.1.

TABLE 7.2.1 COEFFICIENTS OF LATERAL STIFFNESS MATRIX K

i	j	K_{ij}	i	j	K_{ij}
1	1	24656.73984	5	5	19020.09072
2	1	-15447.02672	6	1	-104 .49885
2	2	19398.19648	6	2	379.80768
3	1	6014.77032	6	3	-1460.30870
3	2	-14085.06224	6	4	5636.66408
3	3	19045.44688	6	5	-13986.74816
4	1	1557.82708	6	6	19018.49920
4	2	5662.02064	7	1	27.05810
4	3	-13993.69840	7	2	-98.34438
4	4	19021.77744	7	3	378.12073
5	1	403.47717	7	4	-1459.51298
5	2	-1466.46316	7	5	5635.07272
5	3	5638.35096	7	6	-13980.98720
5	4	-13987.54400	7	7	18996.35488

TABLE 7.2.1 (CONTD.)

i	j	K_{ij}	i	j	K_{ij}
8	1	-6.98053	9	7	5282.92468
8	2	25.37116	9	8	-12621.37136
8	3	-97.54865	9	9	13746.88560
8	4	376.52927	10	1	-0.28300
8	5	-1453.75192	10	2	1.02860
8	6	5612.92848	10	3	-3.95481
8	7	-13895.51440	10	4	15.26522
8	8	18666.35120	10	5	-58.93789
9	1	1.70166	10	6	227.55889
9	2	6.18480	10	7	-878.60471
9	3	23.77970	10	8	3392.29240
9	4	-91.78758	10	9	-5321.64084
9	5	354.38513	10	10	2627.20750
9	6	-1368.27912			

SHEARWALL STRUCTURE TYPE II (METHOD I)

TABLE 7.2.2 FREQUENCIES, TIME PERIODS AND MODE SHAPES

S.No.	1	2	3	4	5
FREQUENCY (rad/sec.)	7.83408	27.74951	59.34451	105.32990	166.10141
TIME PERIOD (Sec)	0.80236	0.22652	0.10592	0.05968	0.03784
	0.01904	-0.06744	0.14116	0.23103	-0.32265
	0.06485	-0.20686	0.36288	0.45180	-0.41138
	0.12499	-0.34251	0.44175	0.27227	0.09382
MODE	0.19108	-0.42088	0.29416	-0.16956	0.43276
	0.25730	-0.41206	0.01028	-0.42802	0.07311
SHAPES	0.31954	-0.31079	-0.30193	-0.23894	-0.40525
	0.37506	-0.13415	-0.41226	0.19596	-0.23516
	0.42233	0.08543	-0.26887	0.41289	0.30386
	0.46119	0.31156	0.07140	0.15473	0.31835
	0.49322	0.51909	0.47760	-0.40806	-0.33470

TABLE 7.2.2 CONTD.

S. No.	6	7	8	9	10
FREQUENCY (rad/sec.)	240.80706	326.69060	417.24629	500.20825	558.88854
TIME PERIOD (Sec)	0.02610	0.01924	0.01506	0.01257	0.01125
M	0.40181	0.45444	0.46377	0.40523	-0.2485
O	0.23321	-0.03352	-0.29404	-0.42746	0.3253
D	-0.39403	-0.38416	-0.04932	0.33776	-0.3881
E	-0.141888	0.34297	0.36152	-0.14060	0.4202
	0.42507	0.11394	-0.42025	-0.10037	-0.4198
S	0.05421	-0.43286	0.18476	0.30997	0.3869
H	-0.43705	0.22843	0.17999	-0.42260	-0.32417
A	0.03144	0.25467	-0.41978	0.40355	0.23649
P	0.40413	-0.41248	0.35543	-0.25284	-0.1282
E	-0.26424	0.19908	-0.14015	0.08765	0.04145
S:					

WIND ANALYSIS (t_r=0.1, F=1.0 Kip)

TABLE 7.2.3 DYNAMIC LOAD FACTORS$(DLF)_{max}$

MODE	t_r Sec	T Sec	t_r/T	$(DLF)_{max}$
1	0.1	0.80236	0.1247	1.98
2	" "	0.22652	0.4620	1.70
3	" "	0.10592	0.945	1.02
4	" "	0.05968	1.675	1.18
5	" "	0.03784	2.645	1.12

WIND ANALYSIS (CONTD.)

TABLE 7.2.4 MODAL STATIC DEFLECTIONS

MODE	1	2	3	4	5
MODAL STATIC DEFLECTIONS	0.25036070	-0.00718893	0.00126887	0.00023751	-0.00009726

TABLE 7.2.5 UPPER AND LOWER BOUNDS OF DEFLECTIONS AND SHEAR FORCES

STOREY		1	2	3	4	5
DEFLECTION BOUND (in)	UPPER	0.01055	0.03532	0.06680	0.10034	0.13272
	LOWER	0.00948	0.03225	0.06210	0.09486	0.12765
S. F. BOUND (kips)	UPPER	25.1586	24.5559	23.0059	21.1161	18.7634
	LOWER	16.5964	16.3124	15.6057	14.5914	13.2742

TABLE 7.2.5 (CONTD.)

STOREY		6	7	8	9	10
DEFLECTION BOUND	UPPER	0.16270	0.18818	0.21090	0.23260	0.25161
	LOWER	0.15845	0.18593	0.20936	0.22865	0.24458
SHAR FORCE BOUND	UPPER	16.2525	13.3348	10.3776	7.2289	3.7592
	LOWER	11.5949	9.7176	7.5376	5.1367	2.4726

EARTHQUAKE ANALYSIS (DAMPING RATIO = 0.00)

TABLE 7.2.6 MODAL PARTICIPATION FACTORS (MPF) AND S_V

MODE	1	2	3	4	5
TIME PERIOD	0.80236	0.22652	0.10592	0.05968	0.03784
MPF	0.35204	-0.04013	0.01255	0.00528	-0.00265
S_V in/Sec	25.60	12.20	14.60	6.90	9.47

TABLE 7.2.7 UPPER AND LOWER VALUES OF DEFLECTIONS AND SHEAR FORCES

STOREY		1	2	3	4	5
DEFLECTION BOUND	UPPER	0.2470	0.7790	1.3874	1.9991	2.5399
	LOWER	0.1770	0.5972	1.1418	1.7352	2.3276
S.F. BOUND	UPPER	1368.311	1287.152	1135.701	1013.7081	862.335
	LOWER	688.932	640.823	562.999	499.112	423.742

STOREY		6	7	8	9	10
DEFLECTION (in) BOUND	UPPER	3.1061	3.5344	3.9199	4.3356	4.8099
	LOWER	2.8843	3.3816	3.8067	4.1591	4.4531
S.F. (Kips) BOUND	UPPER	763.724	704.232	462.094	311.203	181.280
	LOWER	371.931	295.727	224.064	150.659	82.997

TABLE 7.2.7 UPPER AND LOWER VALUES OF DEFLECTIONS AND SHEAR FORCES

STOREY		1	2	3	4	5
DEFLECTION BOUND	UPPER	0.2470	0.7790	1.3874	1.9991	2.5399
	LOWER	0.1770	0.5972	1.1418	1.7352	2.3276
S.F. BOUND	UPPER	1368.311	1287.152	1135.701	1013.7081	862.335
	LOWER	688.932	640.823	562.999	499.112	423.742

STOREY		6	7	8	9	10
DEFLECTION (in) BOUND	UPPER	3.1061	3.5344	3.9199	4.3356	4.8099
	LOWER	2.8843	3.3816	3.8067	4.1591	4.4531
S.F. (Kips) BOUND	UPPER	763.724	704.232	462.094	311.203	181.280
	LOWER	371.931	295.727	224.064	150.659	82.997

EARTHQUAKE ANALYSIS (DAMPING RATIO=0.02)

TABLE 7.2.8 MODAL PARTICIPATION FACTORS (MPF) AND S_v

MODE	1	2	3	4	5
TIME PERIOD	0.80236	0.22652	0.10592	0.05968	0.03784
MPF	0.35206	-0.04013	0.01255	0.00528	-0.00265
S_v in/sec	18.10	10.60	7.50	3.94	3.15

TABLE 7.2.9 UPPER AND LOWER BOUNDS OF DEFLECTIONS (INCHES) AND SHEAR FORCES (KIPS)

STOREY		1	2	3	4	5
DEFLECTION BOUND	UPPER	0.1708	0.5482	0.9902	1.4314	1.8253
	LOWER	0.1255	0.4240	0.8107	1.2309	1.6488
S.F. BOUND	UPPER	769.389	731.335	654.618	581.973	499.468
	LOWER	383.178	363.720	326.862	289.440	250.144

TABLE 7.2.9 (CONTD.)

STOREY		6	7	8	9	10
DEFLECTION	UPPER BOUND	2.2051	2.4917	2.7638	3.0838	3.4198
	LOWER BOUND	2.0406	2.3908	2.6914	2.9418	3.1508
SHEAR FORCE	UPPER BOUND	433.189	345.759	266.535	183.873	106.978
	LOWER BOUND	213.536	173.399	133.059	91.783	50.288

TABLE 7.2.10 STIFFNESSES FOR SHEAR WALL STRUCTURE TYPE 2

i	a_i	b_i
1	0.25614769E+08	0.11664000E+05
2	0.21721500E+05	0.11664000E+05
3	0.25614769E+08	0.97394399E+06
4	0.21721500E+05	0.12150000E+03
5	0.83433474E+04	0.25030043E+06
6	0.49963994E+08	0.25030043E+06
7	0.43321500E+05	0.56868257E+07
8	0.49963994E+08	0.21600000E+05
9	0.43321500E+05	0.56868257E+07
10	0.16686695E+05	0.21600000E+05

DATA

K_1 = 0.11197440E+10 n_1 = 36" E=3000 Kip/in^2

K_2 = 0.11197440E+10 n_2 = 36" G=E/2

K_3 = 0.17496000E+08 l = 120" f=1.2

K_4 = 0.25920000E+07 h = 120"

K_5 = 0.25920000E+07

$$g_1 = \frac{6f\,EI_{w1,y}}{G\,A_{w1}\,h^2}$$

$$g_2 = \frac{6fEI_{w2,y}}{G\,A_{w2}\,h^2}$$

TABLE 7.2.11 COEFFICIENTS OF LATERAL STIFFNESS MATRIX K

i	j	K_{ij}	i	j	K_{ij}
1	1	14161.92029	5	5	11702.15552
2	1	-8340.35120	6	1	-3.50090
2	2	11733.67419	6	2	31.55652
3	1	2491.29269	6	3	-278.02187
3	2	-8065.80731	6	4	2459.77600
3	3	11702.60229	6	5	-8062.31616
4	1	-281.53334	6	6	11702.13379
4	2	2460.22079	7	1	0.46857
4	3	-8062.32037	7	2	-3.49436
4	4	11702.18286	7	3	31.51307
5	1	31.97439	7	4	-278.05943
5	2	-278.04661	7	5	2459.73154
5	3	2459.80087	7	6	-8062.31342
5	4	-8062.29773	7	7	11701.70715

TABLE 7.2.11 (CONTD)

i	j	K_{ij}	i	j	K_{ij}
8	1	0.13492	9	7	2429.41428
8	2	0.58601	9	8	-7808.70850
8	3	--3.36674	9	9	9443.36670
8	4	31.64021	10	1	2.99963
8	5	-277.87999	10	2	3.05296
8	6	2459.51688	10	3	3.09402
8	7	-8058.95941	10	4	3.56992
8	8	11673.59875	10	5	0.53576
9	1	-1.09246	10	6	28.89178
9	2	-1.16037	10	7	-219.99296
9	3	-0.74796	10	8	1983.34276
9	4	-4.70207	10	9	3809.48553
9	5	29.83717	10	10	2002.49969
9	6	-276.17484			

TABLE 7.2.12 FREQUENCIES TIME PERIODS AND MODE SHAPES

S.No.	1	2	3	4	5
FREQUENCY RAD/SEC	6.85357	25.70335	56.72948	98.95149	151.65625
TIME PERIOD SEC.	0.91714	0.24455	0.11080	0.06352	0.04145
MODE SHAPES	-0.01737	-0.07116	0.15338	-0.25252	0.35141
	-0.05836	-0.20948	0.37283	-0.45469	0.39430
	-0.11342	-0.34365	0.44536	-0.25493	-0.12522
	-0.17592	-0.42310	0.29374	0.18684	-0.42838
	-0.24108	-0.4196	-0.01071	0.42961	-0.04504
	-0.30534	-0.32484	-0.30057	0.22849	0.41225
	-0.36611	-0.15488	-0.41060	-0.20405	0.21701
	-0.42175	0.06204	-0.06892	-0.41119	-0.31326
	-0.47163	0.29207	0.06593	-0.14973	-0.30750
	-0.51651	0.50966	0.46545	0.39960	0.33180

TABLE 7.2.12 (CONTD.)

Sl. No.	6	7	8	9	10
FREQUENCY RAD/SEC	211.41850	273.75797	332.62275	381.30605	413.4814
TIME PERIOD SEC.	0.02973	0.02296	0.01890	0.01648	0.0152
MODE SHAPE	0.42949	0.46783	0.44833	0.35928	0.20178
	0.18606	-0.09920	-0.34478	-0.42925	-0.2948
	-0.40913	-0.34778	0.02294	0.37812	0.37396
	-0.09916	0.37247	0.32024	-0.19803	-0.4200
	0.42948	0.06501	-0.42973	-0.04760	0.43074
	0.01947	-0.42152	0.22625	0.27754	-0.4049
	-0.43459	0.25765	0.1452	-0.41471	0.34475
	0.05477	0.22859	-0.40838	0.41481	-0.2558
	0.39561	-0.41163	0.36432	-0.26847	0.14149
	-0.26549	0.20405	-0.14718	0.09505	-0.0464

WIND ANALYSIS (t_r=0.1 Sec. F=1.0 Kip)

TABLE 7.2.13 DYNAMIC LOAD FACTORS $(DLF)_{max}$

MODE	t_r sec.	T Sec.	t_r/T	$(DLF)_{max}$
1	0.1	0.91714	0.109	1.99
2	" "	0.24455	0.408	1.71
3	" "	0.11080	0.903	1.10
4	" "	0.06352	1.575	1.20
5	" "	0.04145	2.418	1.13

TABLE 7.2.14 MODAL STATIC DEFLECTIONS

MODE	1	2	3	4	5
MODAL STATIC DEFLECTIONS	0.32350	-0.00925	0.00140	-0.00027	0.00012

TABLE 7.2.15 UPPER AND LOWER BOUNDS OF DEFLECTIONS (INCHES) AND SHEAR FORCES (KIPS)

STOREY		1	2	3	4	5
DEFLECTION BOUND	UPPER	0.01267	0.04166	0.07924	0.12052	0.16199
	LOWER	0.01124	0.03772	0.07322	0.11345	0.155534
S.F.BOUND	UPPER	25.6732	24.9265	23.3763	21.4239	19.0276
	LOWER	15.6558	15.3169	14.6098	13.5761	12.2615

STOREY		6	7	8	9	10
DEFLECTION BOUND	UPPER	0.20230	0.23887	0.27308	0.30843	0.34147
	LOWER	0.19664	0.23570	0.27151	0.30366	0.33261
S.F.BOUND	UPPER	16.5802	13.5598	10.6382	7.4747	3.9398
	LOWER	10.6687	8.7712	6.7051	5.2958	2.5799

EARTHQUAKE ANALYSIS (DAMPING RATIO=0.00)

TABLE 7.2.16 MODAL PARTICIPATION FACTORS (MPF) AND S_v

MODE	1	2	3	4	5
TIME PERIOD (SEC)	0.91714	0.24455	0.11080	0.06352	0.04145
MPF	0.39717	-0.04741	0.01331	-0.00569	0.00291
S_v inch/sec.	15.75	9.07	17.70	7.10	9.47

TABLE 7.2.17 UPPER AND LOWER BOUNDS OF DEFLECTIONS (INCHES) AND SHEAR FORCES (KIPS)

STOREY		1	2	3	4	5
DEFLECTION BOUND	UPPER	0.1953	0.5722	0.9760	1.3709	1.7093
	LOWER	0.1194	0.3867	0.7324	1.1176	1.5189
S.F.BOUND	UPPER	1214.230	1128.164	982.381	862.109	725.549
	LOWER	637.149	587.025	508.884	439.348	368.777

TABLE 7.2.17 (CONTD.)

STOREY		6	7	8	9	10
DEFLECTION BOUND	UPPER	2.1411	2.4678	2.7538	3.1059	3.5851
	LOWER	1.9165	2.2932	2.6392	2.9530	3.2404
S.F.BOUND	UPPER	652.245	510.990	385.453	254.446	157.159
	LOWER	327.852	257.357	189.312	124.287	75.302

EARTHQUAKE ANALYSIS (DAMPING RATIO = 0.02)

TABLE 7.2.18 MODAL PARTICIPATION FACTORS (MPF) AND S_v

MODE	1	2	3	4	5
TIME PERIOD (SEC)	0.91714	0.24455	0.11080	0.06352	0.04145
MPF	0.39717	-0.04741	0.01331	-0.00569	0.00291
S_v inch/Sec.	11.40	10.40	8.27	3.54	3.15

TABLE 7.2.19 UPPER AND LOWER BOUNDS OF DEFLECTIONS (INCHES) AND SHEAR FORCES (KIPS)

STOREY		1	2	3	4	5
DEFLECTION BOUND	UPPER	0.1389	0.4213	0.7383	1.0452	1.3084
	LOWER	0.0880	0.2868	0.5430	0.8240	1.1110
S.F.BOUND	UPPER	662.428	624.316	553.132	484.402	409.432
	LOWER	334.224	314.411	278.744	241.140	203.856
STOREY		6	7	8	9	10
DEFLECTION BOUND	UPPER	1.5841	1.7853	1.9810	2.2925	2.6523
	LOWER	1.3922	1.6600	1.9101	2.1403	2.3527
S.F.BOUND	UPPER	356.598	280.602	214.372	148.999	91.029
	LOWER	172.619	137.251	103.539	72.231	43.112

CHAPTER VIII

EXPERIMENTAL SET-UP AND DYNAMIC TESTING OF MODELS OF SHEARWALL STRUCTURES :

The dynamic testing of models of two types of shearwall structures was carried out in the Structural Dynamics Laboratory of Indian Institute of Technology, New Delhi.

8.1 EXPERIMENTAL SET-UP :

The main equipment used in the experiment is given below :

1. Vibration Generator (Ling Altec Electromagnetic Type).
2. D-880-A and A/1 2-Phase L.F. Decade Oscillator (Muirhead & Co. Limited).
3. Philips Double Beam Oscilloscope (PM 3230/90) D.C. 10 MH_z.
4. Amplifier Type PP 60 VAP - Ling Altec Type.
5. Voltmeter and Accelerometer.
6. Models of shearwall structures Type I & II.
7. Heavy Concrete Block.

A rig was used to hold the frames which were fixed at the base. The supporting structure consisted of a heavy concrete block with an I-section embedded in it. The model was sandwitched between two 4"x4"x3/8" angles fixed to the flat surface of I-section through high tensile bolts.

8.2 DESCRIPTION OF MODELS :

Two models of shearwall structure Type I and II were fabricated from a perspex sheet 7/16" thick. The dimensioned sketches of the models are shown in Fig. 8.1. and Fig. 8.2 for shearwall structures Type I & II respectively.

$$E \text{ for perspex} = 4.33 \times 10^5 \text{ psi}$$

$$G \text{ for perspex} = 1.6914 \times 10^5 \text{ psi}$$

$$\text{Mass density for perspex} = 1.135 \times 10^{-4} \text{ lb.sec.}^2/\text{in}^4.$$

8.3 DYNAMIC TESTING OF MODELS :

8.3.1 Testing Procedure :

The models were excited harmonically by an electromagnetic vibrator driven through a Power Amplifer by a Muirhead Decade Oscillator. It was assumed that

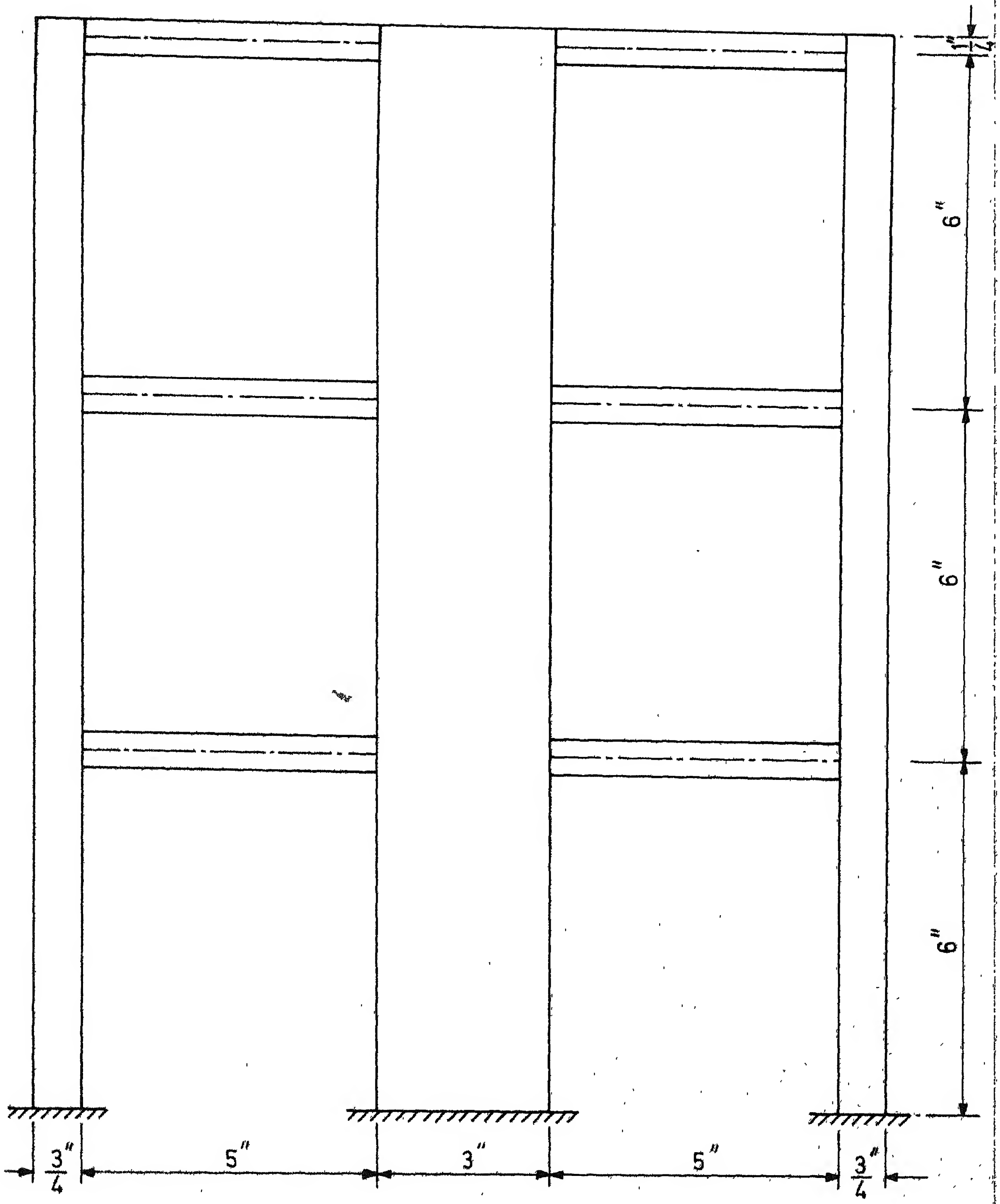


FIG. 8·1 Perspex model of Shearwall structure Type-I

Column size: $\frac{7''}{16} \times \frac{3''}{4}$

Beam size: $\frac{7''}{16} \times \frac{1''}{2}$

Shearwall size: $\frac{7''}{16} \times 3''$

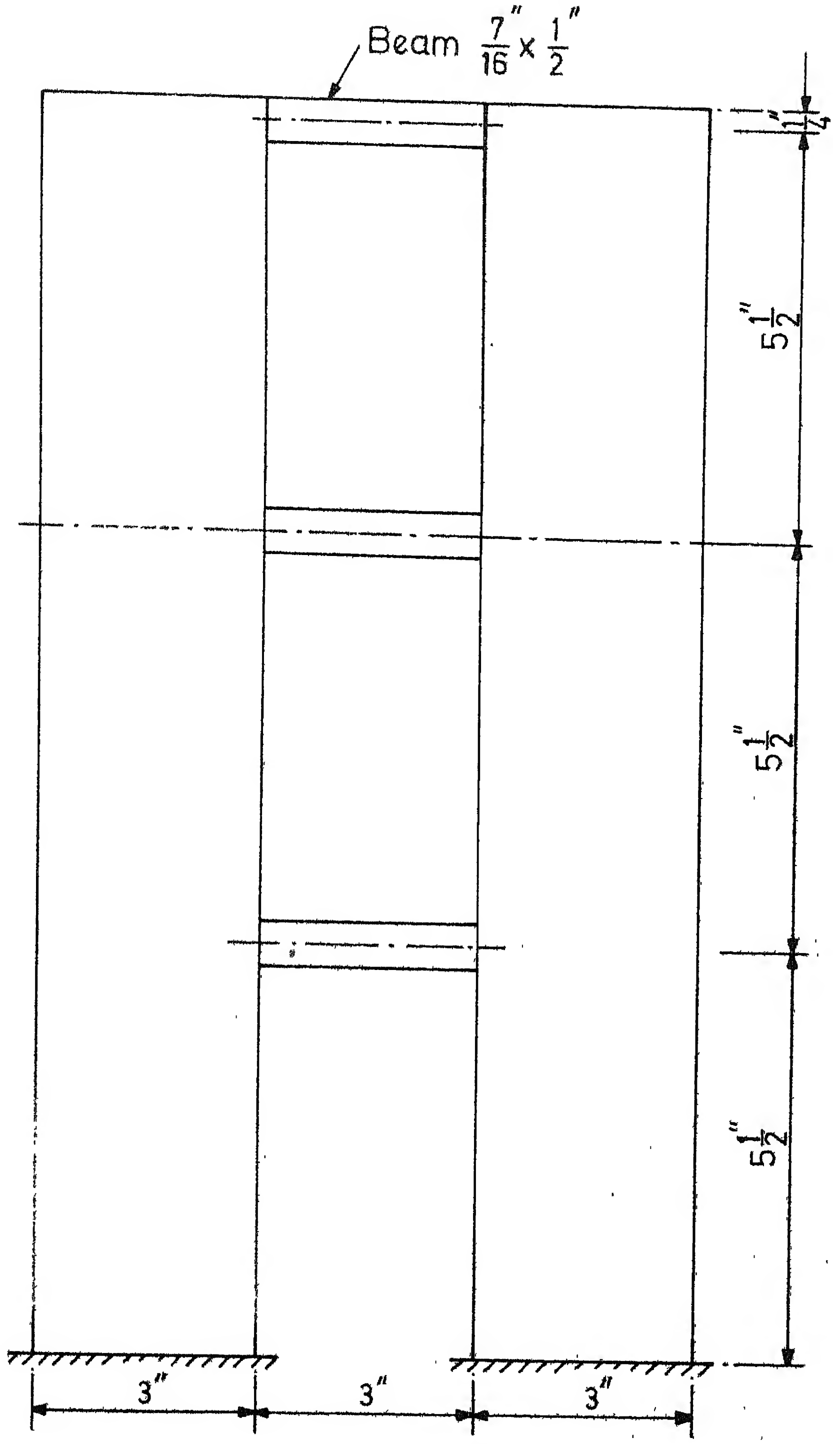


FIG. 8.2. Perspex model of Shearwall structure Type-II

Beam size: $\frac{7}{16}'' \times \frac{1}{2}''$

Shearwall size: $\frac{7}{16}'' \times 3''$

8.2.1 MODEL I OF SHEARWALL STRUCTURE TYPE I

TABLE 8.1 COEFFICIENTS OF LATERAL STIFFNESS MATRIX

i	j	K_{ij} (lbs/in) METHOD I	K_{ij} (lbs/in) METHOD II
1	1	37962.80512	37961.13888
2	1	-21848.02880	-21848.73920
2	2	21760.51456	21749.59776
3	1	5642.18720	5644.77984
3	2	-8217.46520	-8205.04808
3	3	3952.20676	3934.7175

TABLE 8.2 COEFFICIENTS OF DIAGONAL MASS MATRIX

i	1	2	3
m_{ii} (lbs. sec.^{2}/in)	0.001589	0.001589	0.0009745

8.2.2 MODEL II OF SHEARWALL STRUCTURE TYPE II

TABLE 8.3 COEFFICIENTS OF LATERAL STIFFNESS MATRIX

i	j	K_{ij} (lbs/in)	
		METHOD I	METHOD II
1	1	95703.87072	55487.11168
2	1	-55160.46912	-31173.18816
2	2	55141.42656	36506.06816
3	1	14294.75968	7776.83688
3	2	-20923.21520	-14659.09632
3	3	10113.35736	7694.90864

TABLE 8.4 COEFFICIENTS OF DIAGONAL MASS MATRIX

i	1	2	3
m_{ii} lbs.sec^2/in	0.00171314	0.00171314	0.0009683

the force exerted by the vibrator was proportional to the input voltage, the value of which was maintained constant throughout the single observation for a mode. The input current to the vibrator was controlled by an output level knob in the oscillator and its value was generally kept at 1.0 ampere. The input current was also used as a reference signal for the response of the frame.

The vibration set-up is shown diagrammatically in Fig. 8.3. The response signal at the top of the frame was picked up by an accelerometer. The output signal from the oscillator was also fed to the oscilloscope. In the first mode the response from the accelerometer was maximum at the extreme free end of the frame and this was observed to decrease gradually as the accelerometer was moved towards the fixed base. In the second mode the response from the accelerometer was found to be zero at a mode point along the frame while it was zero at two node points in the third mode. The estimate of natural frequencies was obtained by feeding the response from the accelerometer fixed at the top free end to the oscilloscope and by looking at figures on the screen formed by changing the

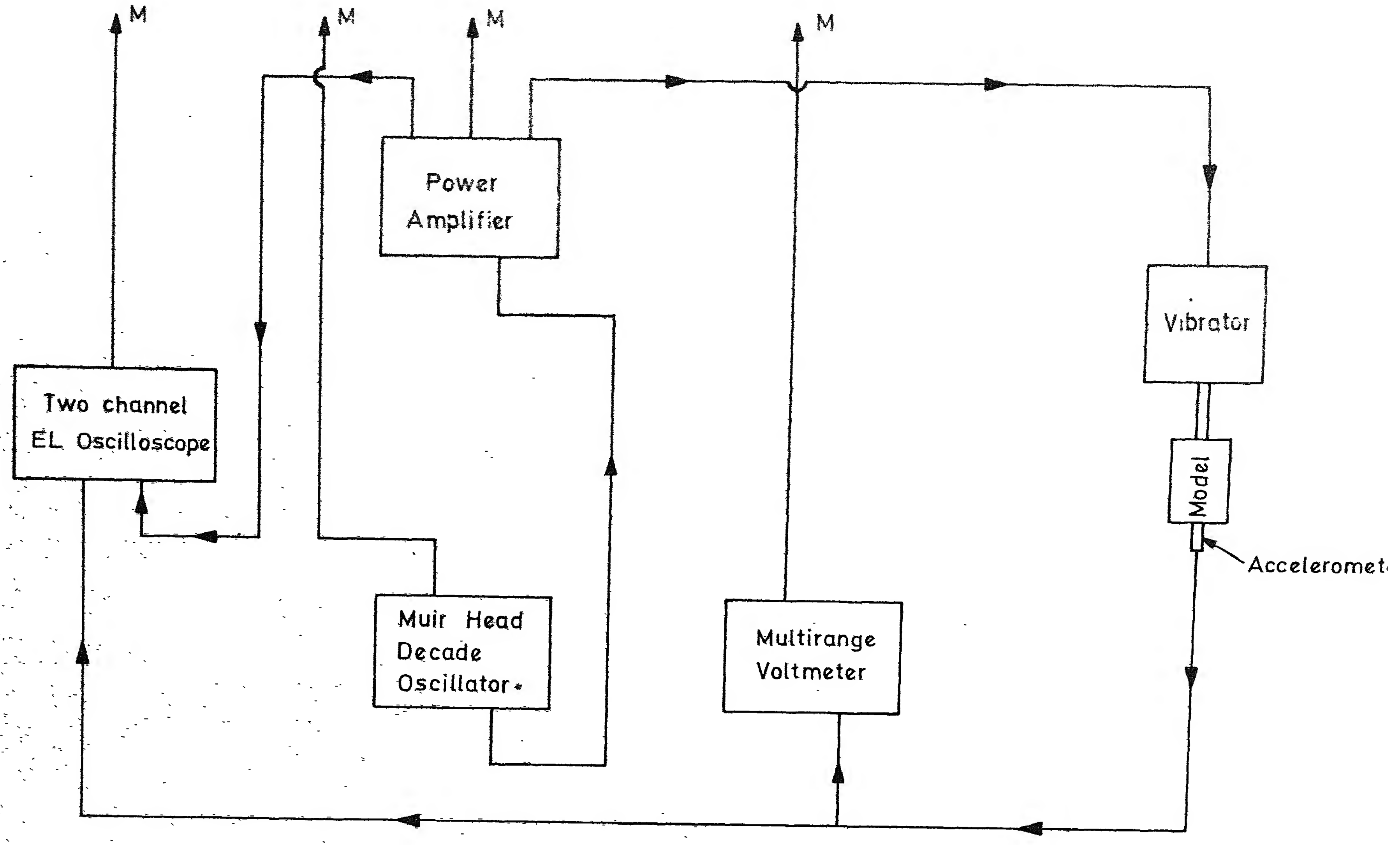


FIG. 8·3. Schematic Diagram of Vibration-set up.

8.4 COMPARISON OR RESULTS

8.4.1 MODEL I

TABLE 8.5 COMPARISON OF NATURAL FREQUENCIES

MODE	THEORY (CPS)		EXPERIMENT (CPS)
	METHOD I	METHOD II	
1	84.20242	83.60950	91.60
2	392.60753	392.07436	412.0
3	944.82902	944.75662	978.0

8.4.2 MODEL II

TABLE 8.6 COMPARISON OF NATURAL FREQUENCIES

MODE	THEORY (CPS)		EXPERIMENT (CPS)
	METHOD I	METHOD II	
1	132.88454	126.83884	132.0
2	611.22852	540.55453	628.0
3	1449.71594	1112.28521	1455.0

8.4 COMPARISON OR RESULTS

8.4.1 MODEL I

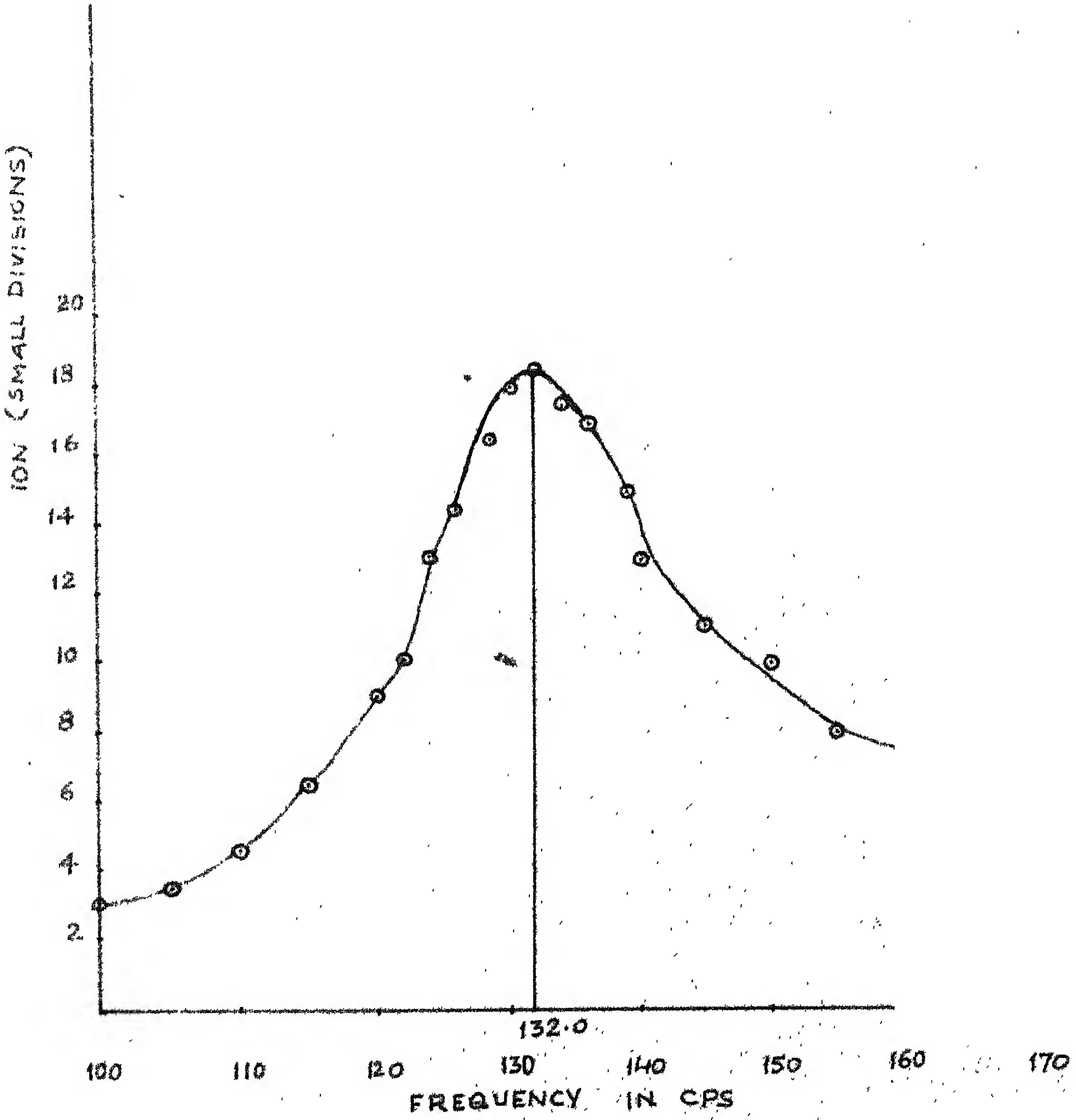
TABLE 8.5 COMPARISON OF NATURAL FREQUENCIES

MODE	THEORY (CPS)		EXPERIMENT (CPS)
	METHOD I	METHOD II	
1	84.20242	83.60950	91.60
2	392.60753	392.07436	412.0
3	944.82902	944.75662	1078.0

8.4.2 MODEL II

TABLE 8.6 COMPARISON OF NATURAL FREQUENCIES

MODE	THEORY (CPS)		EXPERIMENT (CPS)
	METHOD I	METHOD II	
1	132.88454	126.83884	132.0
2	611.22852	540.55453	628.0
3	1449.71594	1112.28521	1455.0



AMPLITUDE PLOT AROUND FIRST FREQUENCY
(MODEL II)

8.4.

frequencies. When the figures appeared to be circles, the frequencies were defined as natural frequencies. A typical plot of amplitude versus frequencies is shown in Fig. 8.4.

8.4 DISCUSSION OF EXPERIMENTAL RESULTS :

The comparison of experimental & theoretical values of natural frequencies of Model I is shown in Table 8.5. The experimental values are higher due to stiffness of joints. The theoretical values were obtained by idealising the model as 3 degree-of-freedom system but actually it is a continuous structure. In case of Model II, the experimental values of natural frequencies are quite comparable with the theoretical values obtained by Method I (Table 8.6). In the Method II, the effects of shear and axial deformations have been considered.

CHAPTER IX

RESULTS AND CONCLUSIONS

The shearwall structures are well suited for construction in earthquake areas and have acted satisfactorily during recent disasters. Shearwalls in building usually become economical as soon as lateral forces govern the design and proportioning of structural components like beam and columns. Buildings upto 70 storeys have been built in U.S.A. using shearwalls.

9.1 SHEARWALL STRUCTURE TYPE I :

The shearwall structure Type I has been analysed by two methods. In the Method I, 4 degrees of freedom per storey were taken. The axial deformations of the columns were included in the Method II, besides the 4 degrees in the first method. The lateral stiffness matrices were formulated in both the above cases. It was found from the results (Chapter VII) that the elements of the lateral stiffness matrix differ by negligible values. In this type of structure identical results i.e.

frequencies and response due to wind and earthquake forces etc. were obtained in both the cases.

Axial deformations of columns will be important if the frame height to width ratio lies between 3 and 4. If axial deformations are negligible, then, it is worthwhile to neglect them, both to reduce the size of the stiffness matrix and to improve the conditioning of the matrix. In a tall buildings, it will save the memory locations in a computer.

9.2 SHEARWALL STRUCTURE TYPE II :

This type of structure has also been analysed by both the methods (Chapter VII). In Method I, 3 degrees of freedom per storey were taken and only flexural effects were considered. In Method II, axial deformations of the walls and the shear effect on them were taken. The second approach causes difference in the elements of the lateral stiffness matrices from those obtained in the first case. The frequencies obtained in first case are in general higher than those in the second case when the frequencies are low, the difference is quite small but the difference becomes large in the higher frequencies (Tables 7.2.24 7.2.12)

Also the response of the structure due to earthquake and wind forces is different in both the cases (Tables 7.2.3 to 7.2.9 & 7.2.13 to 7.2.19) The second method is more realistic with respect to the behaviour of the structure.

9.3 COMPARISON OF METHODS I & II :

As stated in paragraphs 9.1 and 9.2 the method I is approximate as compared to the method II but the former has got certain advantages over the latter. In method I, geometric properties of the members of the structure i.e. lengths and areas of cross-section of columns, walls and beams can be directly varied from storey to storey without changing the computer programme; however in method II the stiffness coefficients have to be modified for any such geometric change. It has been noted (Tables 8.5 and 8.6) that the experimental values of natural frequencies of perspex models I and II are quite comparable with the theoretical values when method I is used to compute the elements of lateral stiffness matrices. The discussion of experimental values of natural frequencies Vs. theoretical values has been given in paragraph 8.4.

9.4 SHEAR FORCE DISTRIBUTION BETWEEN SHEARWALL AND FRAME :

In shearwall structure Type I, as the lateral stiffness of the wall is much larger than those of columns, the lateral force shared by wall is approximately 95% that of the force at the storey level. The rest 5% is taken up by both the columns.

The moment of inertia of shearwall would normally be at least 50 times greater than that of a column.

9.5 SUGGESTION FOR FURTHER WORK :

It is suggested that finite element technique may be used to develop the stiffness and mass matrices of shearwall structures. Then the response of the structure may be found due to dynamic loads like wind, earthquake, blast etc The behaviour in the inelastic range is yet to be investigated in greater detail. Dynamic analysis of Prefabricated Precast Concrete Tall Buildings has also not received much attention in India.

REFERENCES

1. Rathbun, J.C., "Wind Forces on a Tall Building", Transactions, ASCE, V. 105, 1940, pp. 1-41.

2. Ward, H.S., and Crowford, R., "Determination of the Natural Period of Buildings", Bulletin, Seismological Society of America, V. 54, 1964, pp. 1743-1756.

3. Coull, A. and Smith, B.S. "Tall Buildings", The Proceedings of the Symposium on "Tall Buildings" with particular reference to Shearwall Structures, Pergamon Press, Oxford, 1967.

4. "Response of Buildings to Lateral Forces", reported by ACI Committee 442, Journal of ACI, Feb. 1971, No.2, V. 68, pp. 81-106.

5. Sethuratnam, A. and Dayaratnam, P. "State of Art in Shearwall Structures" Department of Civil Engineering, Indian Institute of Technology, Kanpur, India (To be published).

6. Clough, R.W; Wilson, E.L.; and King, I.P., "Structural Analysis of Multistorey Buildings", Journal of Structural Division, Proc. ASCE, Vol. 90, June, 1964, pp. 19-34.

7. Webster, J.A., "The Static and Dynamic Analysis of Orthogonal Structures Composed of Shearwalls and Frames", Tall Buildings, Pergamon Press Limited, London, 1967, pp. 377-399.

8. Jenkins, W.M., and Harrison, T., "Analysis of Tall Buildings with Shearwalls Under Bending and Torsion", Tall Buildings, Pergamon Press Limited, London, 1967, pp. 413-453.

9. Grad, J., and Brebner, M.A., "Eigenvalues and Eigenvectors of a Real General Matrix", Algorithm 343, "Communications of the ACM", Vol. 11/Number 12/December 1968, pp.820-826.

10. Pipes, Louis A., "Matrix Methods for Engineering", Prentice Hall, Inc., Englewood Cliffs, N.J. (1964).

11. Norris, C.H., Hansen, R.J., Holley, M.J., Biggs, J.M., Namyet, S., and Minami, J.K., "Structural Design for Dynamic Loads", McGraw Hill Book Company, New York, 1959, 453 pp.

12. Biggs, J.M., "Introduction to Structural Dynamics", McGraw Hill Book Company, New York, 1964, 374 pp.

13. Rogers, G.L., "Dynamics of Framed Structures", Wiley, New York, 1959.

14. IS: 1893-1971, Criteria for Earthquake Resistant Design of Structures.

15. IS: 4326-1967, Code of Practice for Earthquake Resistant Construction of Buildings.

16. "Earthquake Resisting Buildings", The Concrete Association of India, 1965.

17. Anonymous, "Analysis of Small Reinforced Concrete Buildings for Earthquake Forces", Portland Cement Association, U.S.A., 1955.

18. Benjamin, J.R., "Statically Indeterminate Structures", McGraw-Hill, New York, 1959.

19. Blume, J.A., Newmark, N.M., and Corning, L.H., "Design of Multi-storey Reinforced Concrete Buildings for Earthquake Motions", Portland Cement Association, U.S.A., 1961.

20. Tezcan, Smith S., "Computer Analysis of Plane and Space Structures", Presented at the American Society of Civil Engineers Structural Engineering Conference, Preprint No. 130, October 19-23, 1964.

SELECTED BIBLIOGRAPHY

1. Davenport, A.G., "Wind Loading on Tall Buildings", Proceedings, Symposium on Wind Effects on High-Rise Buildings, Chicago Committee on High-Rise Buildings, March 1970.

2. "Wind Forces on Structures" Transaction, ASCE, V. 126, Part 2, 1961, pp. 1124-1198.

3. Devenport, A.G., "Wind Loads on Structures", Technical Paper No. 88, National Research Council of Canada, Ottawa, Mar. 1960, 81 pp.

4. Clough, R.W., "Dynamic Effects of Earthquakes", Proceedings, ASCE, V. 86, ST4, Apr. 1960, pp. 49-65.

5. Housner, G.W., "Behaviour of Structures During Earthquakes", Proceedings, ASCE, V. 85, EM4, Oct. 1959, pp. 109-129.

6. "Recommended Lateral Force Requirements and Commentary", Structural Engineers Association of California, San Francisco, 1967.

7. Blume, J.A., Newmark, N.M., and Corning, L.H., "Design of Multistorey Reinforced Concrete Buildings for Earthquake Motions", Portland Cement Association, Skokie, Illinois, 1961, 318 pp.

8. Hurty, W.C., and Rubenstein, M.F., "Dynamics of Structures", Prentice-Hall, Inc., Englewood Cliffs, New Jersy, 1964, 455 pp.

9. "Design of Structures to Resist Nuclear Weapons Effects", Manual of Engineering Practice No. 42, American Society of Civil Engineers, New York, 1961, 150 pp.

10. Khan, F.R., and Sbarounis, J.A., "Interaction of Shearwalls and Frames", Proceedings, ASCE, V. 90, ST3, June 1964, pp. 285-335.

11. Clough, R.W., King, I.P., and Wilson, E.L., "Large Capacity Multistorey Frames Analysis Programs", Proceedings, ASCE, V. 89, ST4, Aug. 1963, pp. 179-204.

12. "A Computer Program to Analyze the Dynamic Response of High-Rise Buildings to Nuclear Blast Loading : V. 1, Elastic Analysis., V. 2, Nonlinear Analysis",

Report No. OCD-OS-63-44, Report to the Protective Structures Division, Office of Civilian Defense, by T.Y. Lin and Associates, Oct. 1963.

13. Goumers, A.J., "Computer Programme for Lateral Load Analysis of Multistorey Frames with Shearwalls", Portland Cement Association, 1968.

14. Cardan, B., "Concrete Shearwalls Combined with Rigid Frames in Multistorey Buildings, Subjected to Lateral Loads", ACI Journal Proceedings, V. 58, No.3, Sept. 1961, pp. 299-316.

15. Gould, P.L., "Interaction of Shearwall - Frame Systems in Multistorey Buildings", ACI Journal, Proceedings V. 62, No.1, Jan. 1965, pp. 45-70.

16. Rosenblueth, E., and Holtz, I., "Elastic Analysis of Shearwalls in Tall Buildings", ACI Journal, Proceedings, V. 56, No. 12, June, 1960, pp. 1209-1222.

17. Bradshour, R.R. "Tall Buildings in a Region of High Seismicity, Journal ACI, Sept. 63, p. 1097-1106.

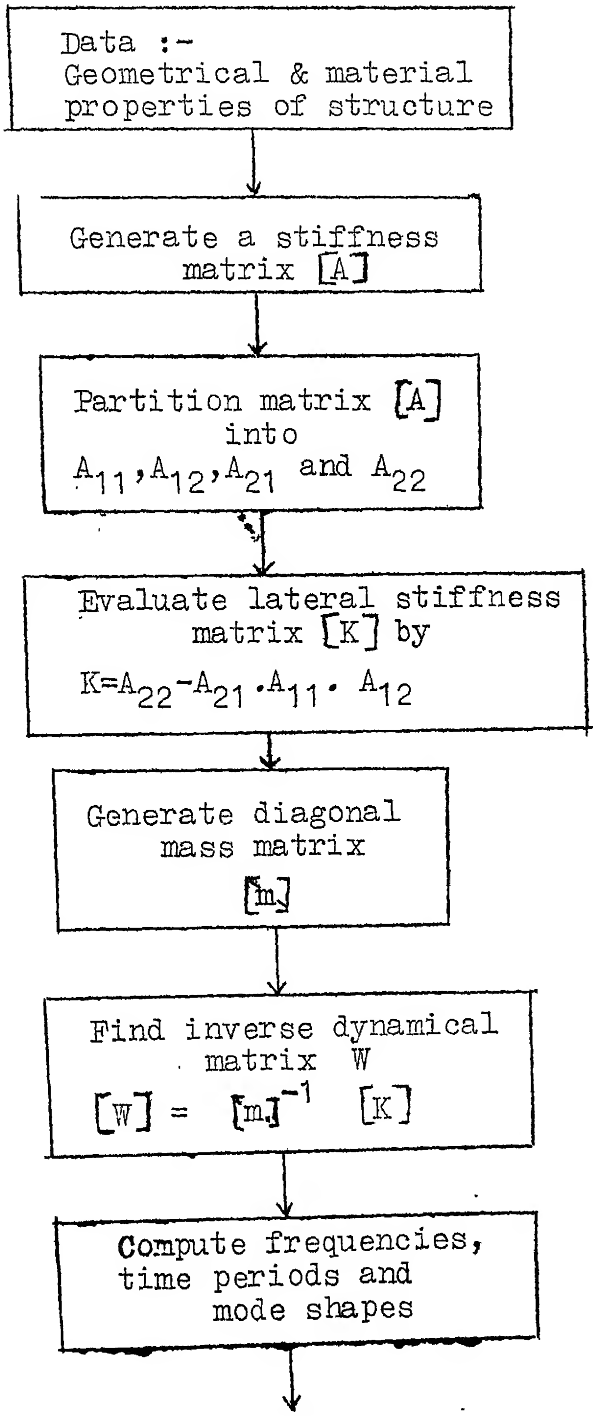
18. Clough, R.W., "Dynamic Effects of Earthquakes", Journal, Structural Division, ASCE, Vol. 86, No. ST4, April, 1960, pages 49.

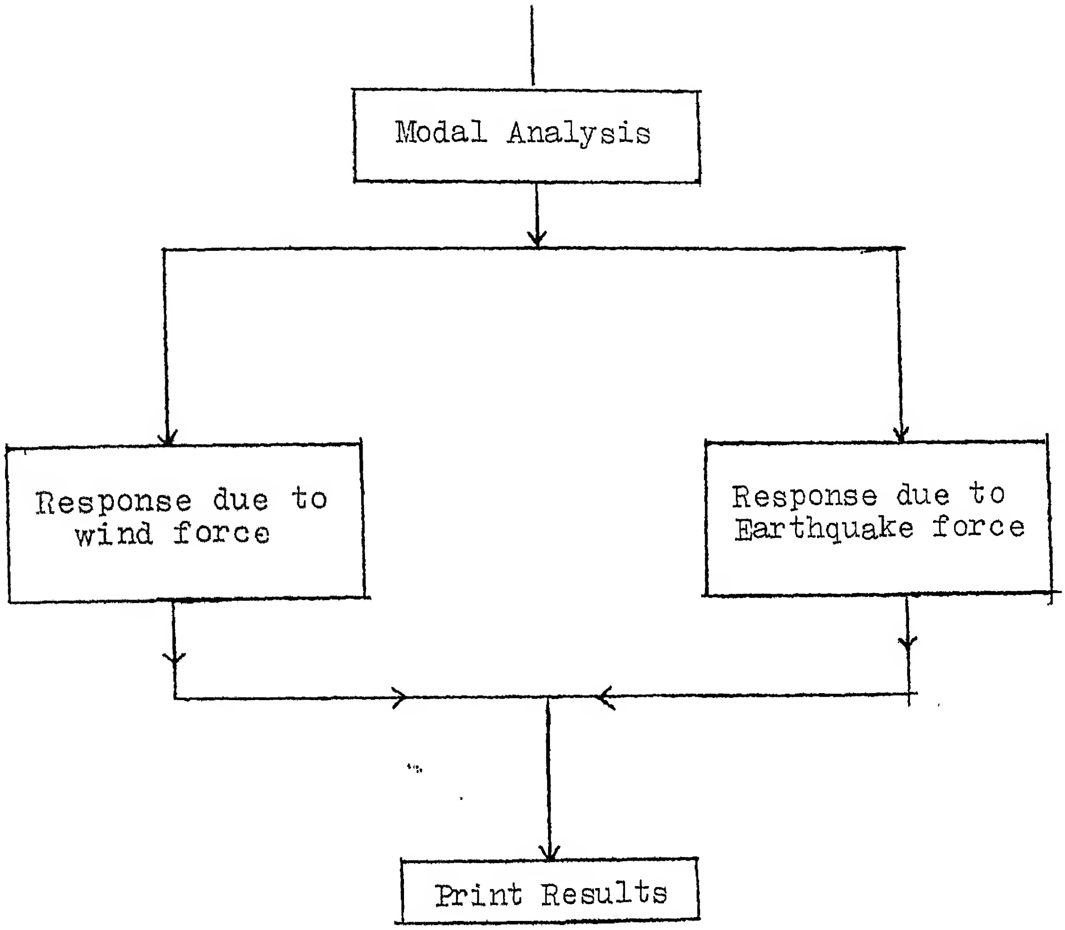
19. Cordan, Bernard, "Concrete Shearwalls Combined With Rogid Frames in Multistorey Buildings Subjected to Lateral Loads", Journal, ACI, Vol. 58, No.3, Sept.,1967.

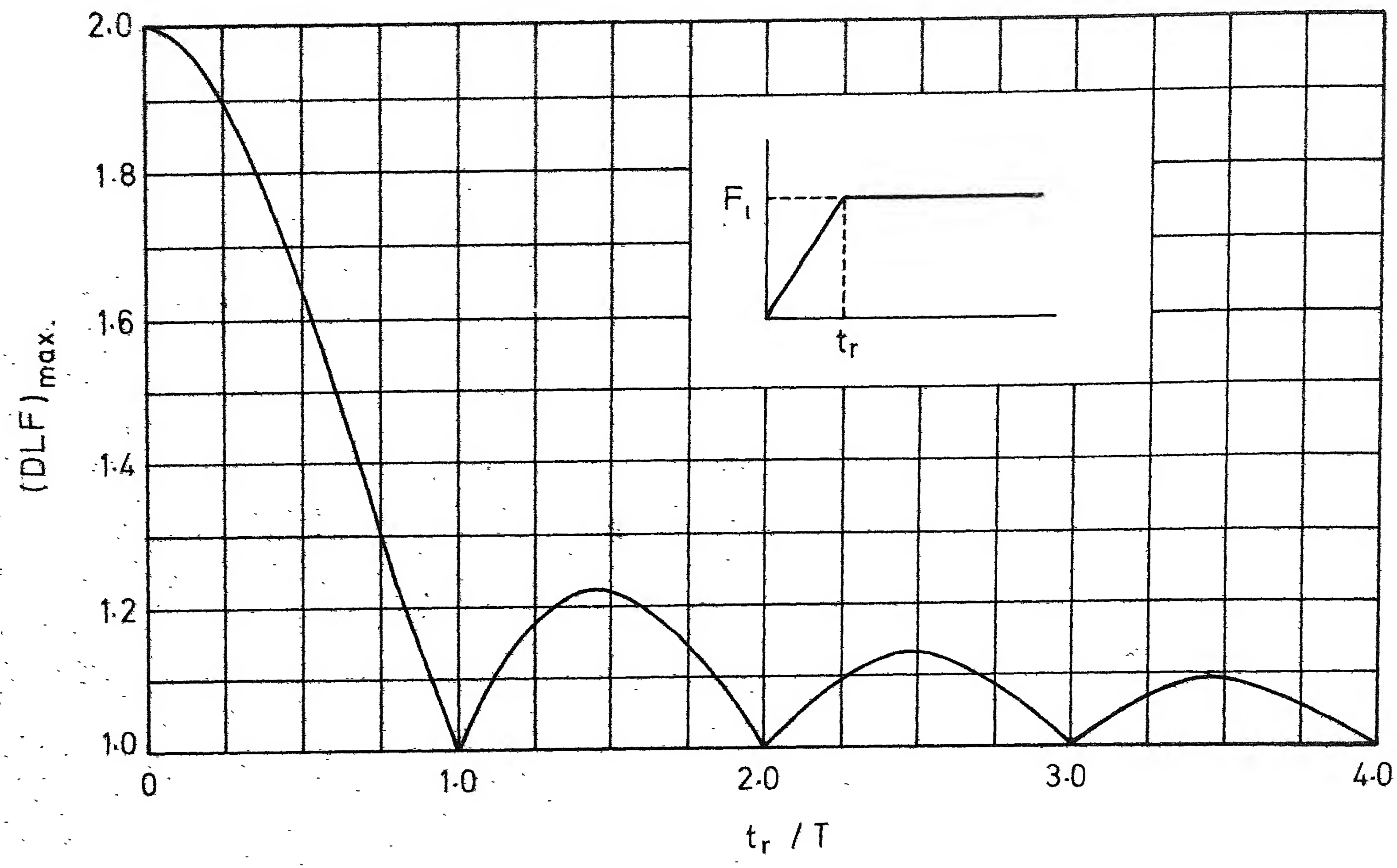
20. Arya, A.S., "Considerations in the Design of Structures Against Earthquake Motions", Inst. of Engnrs. (India), Civil Engg. Div., Journal, July,1971.

APPENDIX

FLOW CHART

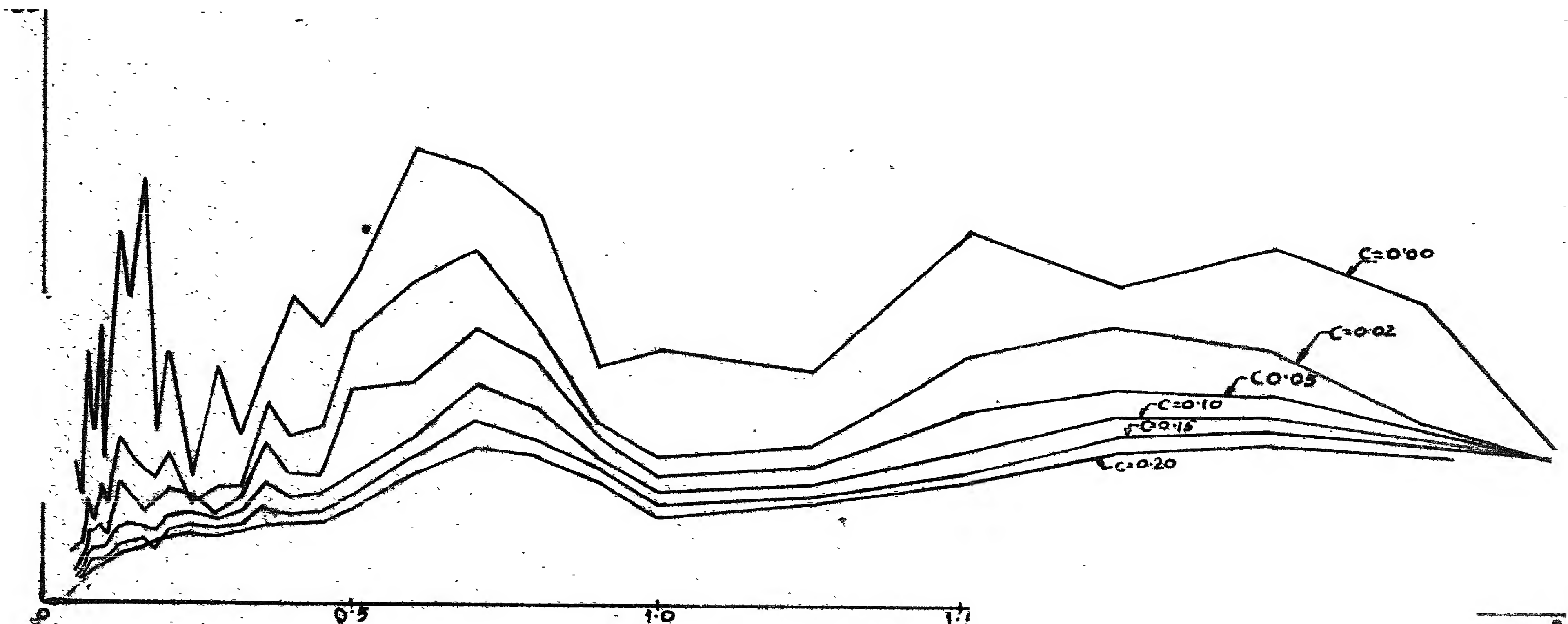

Data :-
Geometrical & material
properties of structure
Generate a stiffness
matrix [A]
Partition matrix [A]
into
A_{11}, A_{12}, A_{21} and A_{22}
Evaluate lateral stiffness
matrix [K] by
$K = A_{22} - A_{21} \cdot A_{11} \cdot A_{12}$
Generate diagonal
mass matrix
[m]
Find inverse dynamical
matrix W
$[W] = [m]^{-1} \; [K]$
Compute frequencies,
time periods and
mode shapes


Modal Analysis
Response due to wind force
Response due to Earthquake force
Print Results



Plot of $(DLF)_{max.}$ Vs. t_r / T.

FIG. A-1.



VELOCITY SPECTRUM

KOYNA EARTHQUAKE DECEMBER 11, 1967, COMPONENT

FIGURE A.2